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PHYSICS for you

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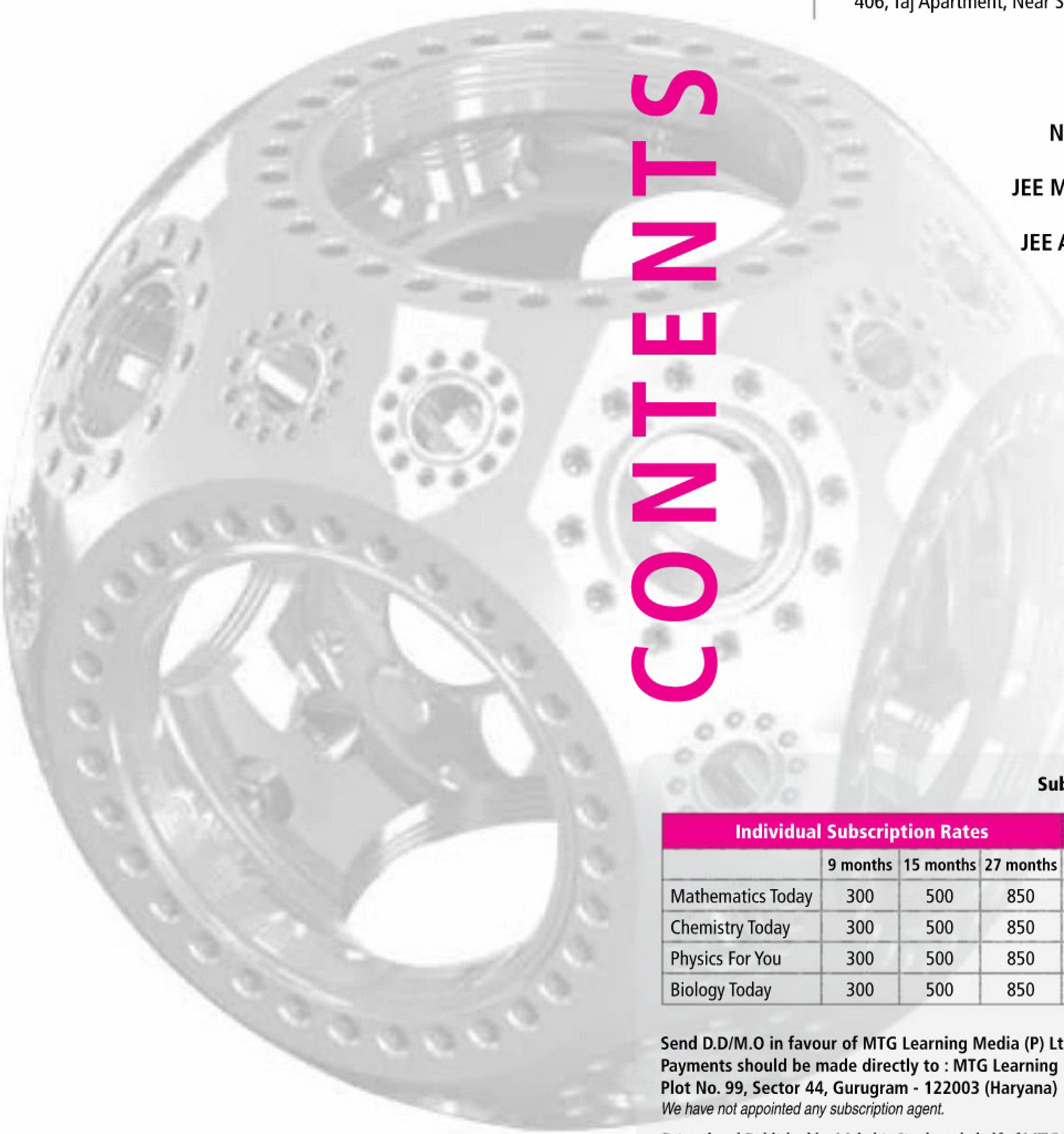
Managing Editor
Mahabir Singh
Editor
Anil Ahlawat

Corporate Office:
Plot 99, Sector 44 Institutional area, Gurugram -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in
Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital, New Delhi - 110029.



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PRACTICE PAPER

NEET

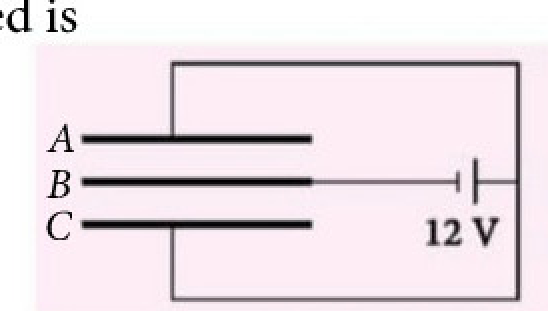
Exam on
13th September 2020

1. A body of mass 1 kg is projected with velocity 50 m s^{-1} at an angle of 30° with the horizontal. At the highest point of its path a force of 10 N acts on body for 5 s vertically upward besides gravitational force. What is the horizontal range of the body? ($g = 10$ m s^{-2})
(a) $125\sqrt{3}$ m (b) $200\sqrt{3}$ m
(c) 500 m (d) $250\sqrt{3}$ m

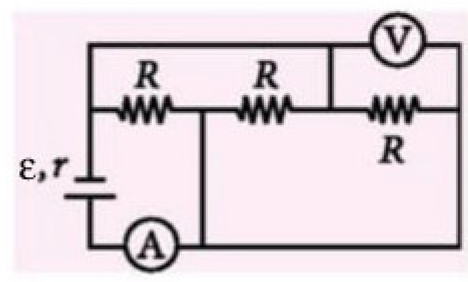
2. The kinetic energy K of a particle moving along a circle of radius R depends upon the distance s as $K = as^2$, where a is a constant. The force acting on the particle is
(a) $2a\frac{s^2}{R}$ (b) $2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$
(c) $2as$ (d) $2a$

3. A hemispherical bowl of radius R is kept on a horizontal table. A small sphere of radius $r(r << R)$ is placed at the highest point at the inside of the bowl and let go. The sphere rolls without slipping. Its velocity at the lowest point is
(a) $\sqrt{5gR/7}$ (b) $\sqrt{3gR/2}$
(c) $\sqrt{4gR/3}$ (d) $\sqrt{10gR/7}$

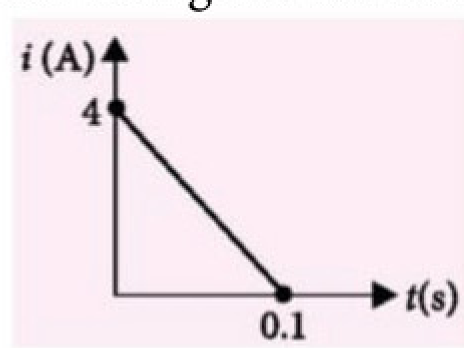
4. Three plates A, B, C each of area 50 cm^2 have separation 3 mm between A and B and 3 mm between B and C. The energy stored when the plates are fully charged is
(a) 1.6×10^{-9} J
(b) 2.1×10^{-9} J
(c) 5×10^{-9} J
(d) 7×10^{-9} J



5. In the circuit shown in the figure, ammeter and voltmeter are ideal. If ε = 4 V, $R = 9\ \Omega$ and $r = 1\ \Omega$, then readings of ammeter and voltmeter are
(a) 1 A, 3 V
(b) 2 A, 3 V
(c) 1 A, 4 V
(d) 4 A, 4 V



6. Some magnetic flux is changed through a coil of resistance 10 Ω. As a result an induced current is developed in it, which varies with time as shown in the figure. The magnitude of change in the flux through the coil (in Wb) is
(a) 2
(b) 4
(c) 6
(d) none of these



7. An AC source is connected with a resistance (R) and an uncharged capacitance (C), in series. The potential difference across the resistor is in phase with the initial potential difference across the capacitor for the first time at the instant (assume that at $t = 0$, emf is zero).
(a) $\frac{\pi}{4\omega}$ (b) $\frac{2\pi}{\omega}$ (c) $\frac{\pi}{2\omega}$ (d) $\frac{3\pi}{2\omega}$

8. The bob of a simple pendulum of length 1.2 m has a velocity of 7 m s^{-1} when it is at the lowest point. The bob would leave the circular path above the centre at a height
(a) 1.0 m (b) 0.867 m
(c) 0.652 m (d) 0.512 m

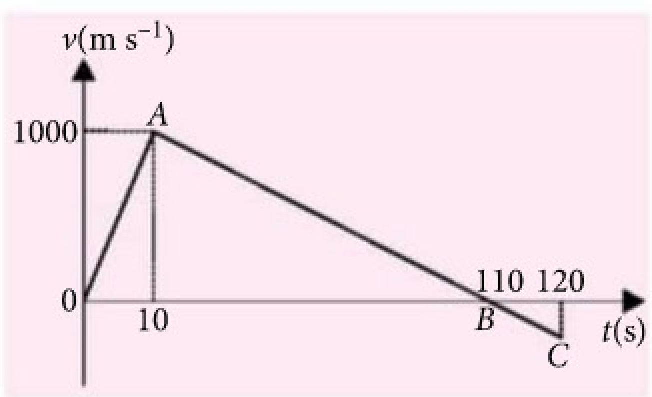
9. What is the height at which the weight of body will be the same as at the same depth from the surface of the earth? Let radius of earth be R.
(a) $\frac{R}{2}$ (b) $\sqrt{5}R - R$
(c) $\frac{\sqrt{5}R - R}{2}$ (d) $\frac{\sqrt{3}R - R}{2}$

10. A steel wire of length 20 cm and uniform cross sectional area 1 mm^2 is tied rigidly at both the ends. The temperature of the wire is altered from 20°C to 40°C. Coefficient of linear expansion of steel is $\alpha = 1.1 \times 10^{-5}$ per °C and Y for steel is 2.0×10^{11} N m^{-2}; the tension in wire is
(a) 2.2×10^6 N (b) 16 N
(c) 8 N (d) 44 N

11. The dimensional formula of a physical quantity is $[M^{-1} L^{-3} T^2]$. The respective errors in measuring M, L, T are 2%, 3%, 4%. The maximum percentage error in measuring the quantity is
(a) 2% (b) 9% (c) 8% (d) 19%

12. A bird moves with velocity 20 m s^{-1} in a direction making an angle of 60° with the eastern line and 60° with vertical upward line. The velocity vector of bird in cartesian coordinates is
(a) $10\hat{i}+10\hat{j}+10\hat{k}$ (b) $10\hat{i}+5\sqrt{2}\hat{j}+10\hat{k}$
(c) $10\hat{i}+10\sqrt{2}\hat{j}+10\hat{k}$ (d) $5\hat{i}+5\sqrt{2}\hat{j}+10\hat{k}$

13. The given graph shows the variation of velocity of a rocket with time. Find the time of burning of fuel from the graph.



(a) 10 s (b) 110 s
(c) 120 s
(d) cannot be estimated from the graph

14. In Young's double slit experiment, the y-coordinates of central maxima and 10^{th} maxima are 2 cm and 5 cm respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5, the corresponding y-coordinates will be
(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm
(c) 2 cm, 4 cm (d) 4/3 cm, 10/3 cm

15. Light of wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy which requires voltage V_0 to prevent them from reaching a collector. In the same setup, light of wavelength 220 nm, ejects electrons which require twice the voltage V_0 to stop them from reaching a collector. The numerical value of voltage V_0 is
(a) $\frac{16}{15}$ V (b) $\frac{15}{16}$ V (c) $\frac{15}{8}$ V (d) $\frac{8}{15}$ V

16. Two identical magnetic dipoles of magnetic moment 2 A m^2 are placed at a separation of 2 m with their axes perpendicular to each other in air. The resultant magnetic field at a midpoint between the dipoles is
(a) $4\sqrt{5}\times10^{-5}$ T (b) $2\sqrt{5}\times10^{-5}$ T
(c) $4\sqrt{5}\times10^{-7}$ T (d) $2\sqrt{5}\times10^{-7}$ T

17. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, the image formed is virtual. If the sizes of the image formed are equal then focal length of the lens will be
(a) 21 cm (b) 11 cm (c) 15 cm (d) 17 cm

18. A charged oil drop falls with terminal velocity v_0 in the absence of electric field. An electric field E keeps it stationary. The drop acquires charge $3q$, it starts moving upwards with velocity v_0. The initial charge on the drop is
(a) $\frac{q}{2}$ (b) q (c) $\frac{3q}{2}$ (d) $2q$

19. Which one of the following statements regarding photo-emission of electrons is correct?
(a) Kinetic energy of electrons increases with the intensity of incident light.
(b) Electrons are emitted when the wavelength of the incident light is above a certain threshold wavelength.
(c) Photoelectric emission is instantaneous with the incidence of light.
(d) Photo electrons are emitted whenever a gas is irradiated with ultraviolet light.

20. A ball of mass m is thrown vertically upwards. Another ball of mass $2m$ is thrown at angle θ with the vertical. Both of them stay in air for the same period of time. The heights attained by the two balls are in the ratio of
(a) 2 : 1 (b) $\cos\theta$: 1
(c) 1 : $\cos\theta$ (d) 1 : 1

21. A block of mass 0.5 kg is moving with a speed of 2.0 m s^{-1} on a smooth surface. It strikes another mass of 1.0 kg at rest and then they move together as a single body. The energy loss during the collision is
(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

22. The temperature of equal masses of three different liquids A, B and C are 12 °C, 19 °C and 28 °C respectively. The final temperature when A and B are mixed is 16 °C and when B and C are mixed is 23 °C.

The final temperature when A and C are mixed is
(a) 18.2 °C (b) 22.2 °C
(c) 20.2 °C (d) 24.2 °C

23. The specific heat of the mixture of two gases at constant volume is 13 R/6. The ratio of the number of moles of the first gas to the second is 1 : 2. The respective gases may be
(a) He, Ne (b) He, N_2 (c) N_2, O_2 (d) N_2, He

24. A spring has force constant K and a mass m is suspended from it. The spring is cut in two parts in the ratio 1 : 3, and the same mass is suspended from the larger part. If the frequency of oscillation in the first case is υ, then the frequency in the second case will be
(a) 2υ (b) 3υ (c) $\frac{2\upsilon}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}\upsilon$

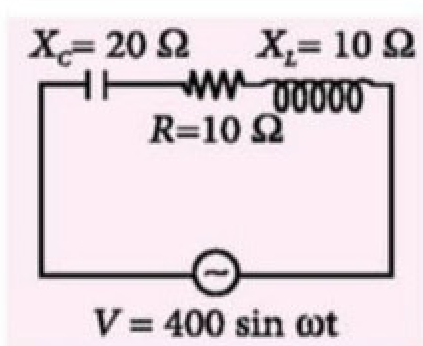
25. A string is under tension so that its length is increased by $\frac{1}{n}$ times its original length. The ratio of fundamental frequency of longitudinal vibrations and transverse vibrations will be
(a) $1 : n$ (b) $n^2 : 1$ (c) $\sqrt{n} : 1$ (d) $n : 1$

26. The driver of a car travelling with speed 30 m s^{-1} towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m s^{-1}, the frequency of reflected sound as heard by driver is
(a) 500 Hz (b) 550 Hz
(c) 555.5 Hz (d) 720 Hz

27. When forward bias is applied to a p-n junction, then what happens to the potential barrier $V_{B'}$ and the width of charge depleted region x ?
(a) V_B increases, x decreases
(b) V_B decreases, x increases
(c) V_B increases, x increases
(d) V_B decreases, x decreases

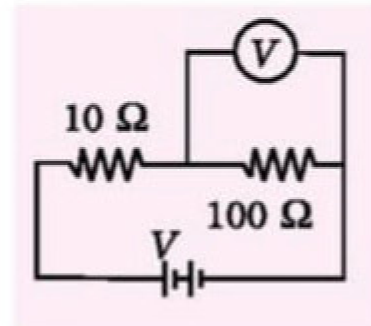
28. The half-life of radium is 1620 yr and its atomic weight is 226 g mol^{-1} The number of atoms that will decay from its 1 g sample per second will be (Avogadro's number $N = 6.023 \times 10^{23}$ atom per mole)
(a) 3.61×10^{10} Bq (b) 3.61×10^{12} Bq
(c) 3.11×10^{15} Bq (d) 31.1×10^{15} Bq

29. Which of the following statements is incorrect regarding the given LCR circuit ?



(a) Voltage will lead the current.
(b) rms value of current is 20 A.
(c) Power factor of the circuit is $\frac{1}{\sqrt{2}}$.
(d) Voltage drop across resistance is 200 V.

30. The potential difference across 100 Ω resistance in the circuit is measured by a voltmeter of 900 Ω resistance. The percentage error in reading the potential difference is



(a) $\frac{10}{9}$
(b) 0.1
(c) 1.0
(d) 10.0

31. A charge q is moving with a velocity $v_1 = 1\hat{i}$ m s^{-1} at a point in a magnetic field and experiences a force $\mathrm{F} = q[-\hat{j} + \hat{k}]$ N. If the charge is moving with a velocity $v_2 = 1\hat{j}$ m s^{-1} at the same point, it experiences a force $\mathrm{F}_2 = q(\hat{i} - \hat{k})$ N. The magnetic induction at that point is
(a) $(\hat{i} + \hat{j} + \hat{k})$ Wb m^{-2} (b) $(\hat{i} - \hat{j} + \hat{k})$ Wb m^{-2}
(c) $(-\hat{i} + \hat{j} - \hat{k})$ Wb m^{-2} (d) $(\hat{i} + \hat{j} - \hat{k})$ Wb m^{-2}

32. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the masses is (g/8), then the ratio of masses is
(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3

33. The velocity of a body moving in a vertical circle of radius r is $\sqrt{7gr}$ at the lowest point of the circle. What is the ratio of maximum and minimum tension ?
(a) 4 : 1 (b) $\sqrt{7} : 1$ (c) 3 : 1 (d) 2 : 1

34. A coin of mass m and radius r having moment of inertia I about the axis passes through its centre and perpendicular to its plane. It is beaten uniformly to form a disc of radius 2r. What will be the moment of inertia about the same axis?
(a) I (b) $2I$ (c) $4I$ (d) $16I$

35. A spherical soap bubble of radius 1 cm is formed inside another bubble of radius 3 cm. The radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is
(a) 0.75 cm (b) 0.75 m
(c) 7.5 cm (d) 7.5 m

36. A heat engine has an efficiency η. Temperatures of source and sink are each decreased by 100 K. Then, the efficiency of the engine
(a) increases (b) decreases
(c) remains constant (d) becomes 1

37. Two masses m_1 and m_2 are suspended together by a massless spring of force constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillation is
(a) $\frac{m_1 g}{k}$ (b) $\frac{m_2 g}{k}$
(c) $\frac{(m_1+m_2)g}{k}$ (d) $\frac{(m_1-m_2)g}{k}$

38. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.00 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is
(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

39. The instantaneous magnitudes of the electric field (E) and the magnetic field (B) vectors in an electromagnetic wave propagating in vacuum are related as
(a) $E = \frac{B}{c}$ (b) $E = cB$
(c) $E = \frac{B}{c^2}$ (d) $E = c^2B$

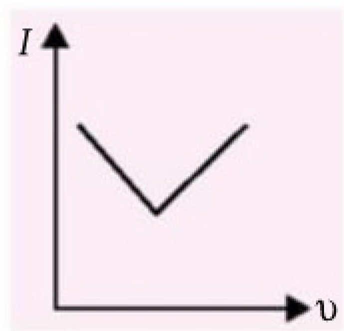
40. The threshold frequency for a photosensitive metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on this metal, the cut-off voltage for the photoelectron emission is nearly
(a) 1 V (b) 2 V (c) 3 V (d) 5 V

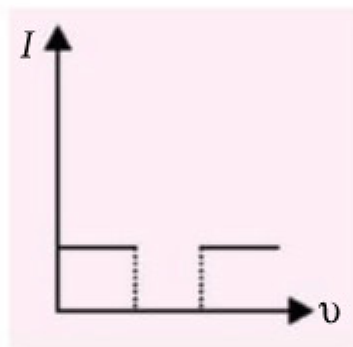
41. Half life of a radioactive substance A is two times the half life of another radioactive substance B. Initially, the number of nuclei of A and B are N_A and N_B respectively. After three half lives of A, number of nuclei of both are equal. Then the ratio $\frac{N_A}{N_B}$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

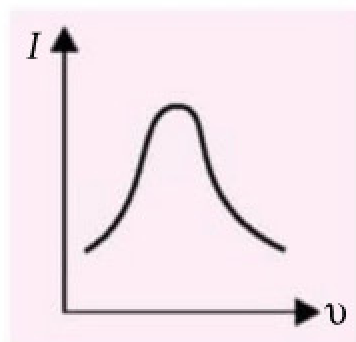
42. The transition in He^+ ion that will give rise to a spectral line having the same wavelength as that of some spectral line in hydrogen atom is
(a) $n = 3$ to $n = 1$ (b) $n = 3$ to $n = 2$
(c) $n = 4$ to $n = 2$ (d) $n = 4$ to $n = 3$

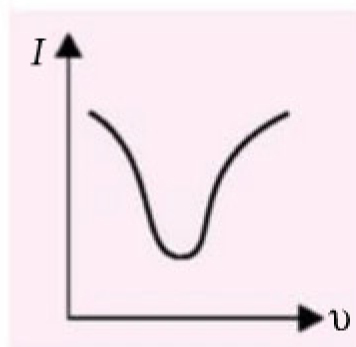
43. One way in which the operation of an n-p-n transistor differs from that of a p-n-p transistor is that
(a) the emitter junction is reverse biased in the n-p-n and forward biased in the p-n-p
(b) the emitter injects minority carriers into the base region of the p-n-p and majority carriers in the base region of the n-p-n
(c) the emitter injects holes into the base region of the p-n-p and electrons into the base region of the n-p-n
(d) the emitter injects electrons into the base region of the p-n-p and holes into the base region of the n-p-n.

44. An AC source of variable frequency υ is connected to an L-C-R series circuit. Which one of the graphs in figure represents the variation of current I in the circuit with frequency υ ?

(a)


(b)


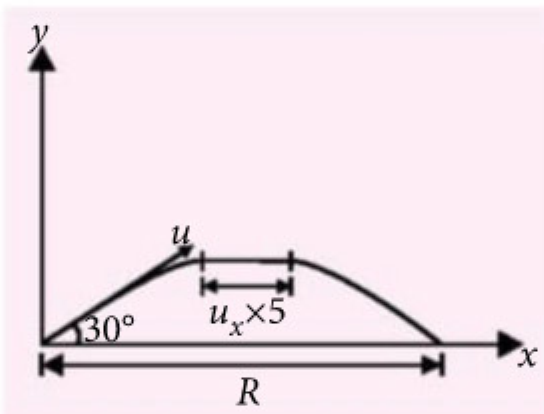
(c)


(d)


45. The insulation property of air breaks down at $E = 3 \times 10^6$ V m^{-1}. The maximum charge that can be given to a sphere of diameter 5 m is approximately (in coulombs)
(a) 2×10^{-2} (b) 2×10^{-3}
(c) 2×10^{-4} (d) 2×10^{-5}

SOLUTIONS

1. **(d):** For 5 s, weight of the body is balanced by the given force. Hence, it will move in a straight line as shown in the figure.



$$R = \frac{u^2 \sin 2\theta}{g} + (u\cos\theta)(5)$$

$$= \frac{(50)^2 \cdot \sin 60^\circ}{10} + (50 \times \cos 30^\circ)(5) = 250\sqrt{3}\text{ m}$$

2. (b): Here, $K = \frac{1}{2}mv^2 = as^2$ $\therefore$ $mv^2 = 2as^2$

Differentiating with respect to time t

$$2mv\frac{dv}{dt} = 4as\frac{ds}{dt} = 4asv \Rightarrow m\frac{dv}{dt} = 2as$$

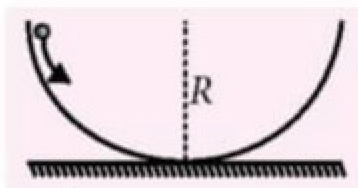
$\therefore$ Tangential force, $F_t = 2as$

Centripetal force, $F_c = \frac{mv^2}{R} = \frac{2as^2}{R}$

$\therefore$ Net force acting on the particle

$$F = \sqrt{F_t^2 + F_c^2} = \sqrt{(2as)^2 + \left(\frac{2as^2}{R}\right)^2} = 2as\sqrt{1 + s^2/R^2}$$

3. (d): As, it is clear from figure, on reaching the bottom of the bowl, loss in potential energy = mgR, and gain in kinetic energy



$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

As, gain in KE = loss in PE

$$\therefore \quad \frac{7}{10}mv^2 = mgR$$

$$\Rightarrow \quad v = \sqrt{\frac{10\,gR}{7}}$$

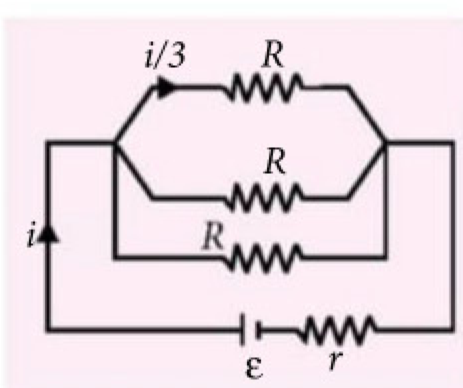
4. (b): Here circuit is equivalent to two capacitors in parallel.

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$

$$\therefore \quad \text{Energy stored} = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\left(\frac{2\varepsilon_0 A}{d}\right)V^2$$

$$= \frac{8.86\times10^{-12}\times50\times10^{-4}\times12\times12}{3\times10^{-3}} = 2.1\times10^{-9}\text{ J}$$

5. (a): The given three resistors are in parallel



$$\therefore \quad i = \frac{\varepsilon}{r + R/3} = \frac{4}{1+3} = 1\text{A}$$

$$V = \frac{i}{3}R = \left(\frac{1}{3}\right)(9) = 3\text{V}$$

6. (a): Charge induced in coil

$dq = \frac{d\phi}{R} = i\,dt$ = Area under i-t graph

$\therefore$ $d\phi$ = (Area under i-t graph) R

$$= \frac{1}{2}\times4\times0.1\times10 = 2\text{ Wb}$$

7. (d): Let $V = V_0 \sin\omega t$ (as $V = 0$ at $t = 0$)

Then $V_R = V_0 \sin\omega t$

and $V_C = V_0 \sin(\omega t - \pi/2)$

V and V_R are in same phase. While V_C lags V (or V_R) by 90°. Now V_R is in same phase with initial potential difference across the capacitor for the first time when,

$$\omega t = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$$

$$\therefore \quad t = \frac{3\pi}{2\omega}$$

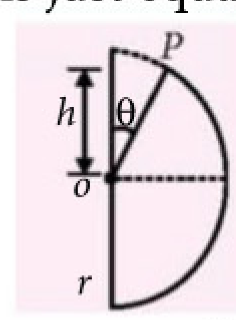
8. (b): Here, $r = l = 1.2$ m, $u = 7$ m s^{-1}

Now, $\sqrt{2gr} = \sqrt{2\times9.8\times1.2} = 4.85\text{ m s}^{-1}$

and $\sqrt{5gr} = \sqrt{5\times9.8\times1.2} = 7.67\text{ m s}^{-1}$

The condition $\sqrt{2gr} < u < \sqrt{5\,gr}$ is satisfied.

Therefore, the bob will leave the vertical circle at P, where component of weight along PO is just equal to centripetal force (mv^2/r).



i.e., $mg\cos\theta = \frac{mv^2}{r}$

or $v^2 = rg\cos\theta = gh$ ...(i)

Height of P above the bottom = $r + h$

At P, loss in KE = gain in PE

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mg(r+h)$$

or $u^2 - v^2 = 2g(r + h)$

using (i), $u^2 = gh + 2gh + 2gr = g(3h + 2r)$

$$h = \frac{1}{3}\left[\frac{u^2}{g} - 2r\right] = \frac{1}{3}\left[\frac{7^2}{9.8} - 2\times1.2\right] = 0.867\text{ m}$$

9. (c): Weight of a body at height h = weight of the body at depth h

$$\frac{gR^2}{(R+h)^2} = g\left(1 - \frac{h}{R}\right)$$

or $\left(1 - \frac{h}{R}\right)\left(1 + \frac{h^2}{R^2} + \frac{2h}{R}\right) = 1$

or $\frac{h^3}{R^3}+\frac{h^2}{R^2}-\frac{h}{R}=0$ or $\frac{h}{R}\left(\frac{h^2}{R^2}+\frac{h}{R}-1\right)=0$

or $\frac{h}{R}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{\sqrt{5}-1}{2}$ or $h=\frac{\sqrt{5}R-R}{2}$

10. (d): Increase in length due to rise in temperature, $\Delta L=\alpha L\,\Delta T$

As, $Y=\frac{FL}{A\Delta L}$;

so $F=\frac{YA\,\Delta L}{L}=\frac{YA\times\alpha L\,\Delta T}{L}=YA\,\alpha\,\Delta T$

$\therefore\ F=2\times10^{11}\times10^{-6}\times1.1\times10^{-5}\times20=44\,\text{N}$

11. (d): Here, $x=[\text{M}^{-1}\,\text{L}^{-3}\,\text{T}^2]$

$$\frac{\Delta x}{x}=\frac{\Delta M}{M}+3\left(\frac{\Delta L}{L}\right)+2\left(\frac{\Delta T}{T}\right)$$

$\therefore$ % error in the quantity x

= 2% + 3 (3%) + 2 (4%) = 19%

12. (c) : Let eastern line be x-axis, northern line as y-axis and vertical upward line as z-axis. Let $\vec{v}$ make angles α, β and γ with x, y and z-axis respectively. Then $\alpha=60°$, $\gamma=60°$

As $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\therefore\ \cos^2 60°+\cos^2\beta+\cos^2\gamma=1$

or $(1/2)^2+\cos^2\beta+(1/2)^2=1$ or $\cos\beta=\frac{1}{\sqrt{2}}$

Now $\vec{v}=(v\cos\alpha)\hat{i}+(v\cos\beta)\hat{j}+(v\cos\gamma)\hat{k}$

$=(20\cos60°)\hat{i}+(20\times1/\sqrt{2})\hat{j}+(20\cos60°)\hat{k}$

$=10\hat{i}+10\sqrt{2}\hat{j}+10\hat{k}$

13. (a) : When the fuel is burning, velocity of the rocket is increasing. After the fuel is exhausted, velocity starts decreasing. From the graph, time of burning of fuel = 10 s.

14. (c) : Fringe width, $\beta=\frac{D\lambda}{d}$, *i.e.*, $\beta\propto\lambda$

So, wavelength λ and hence fringe width β decreases 1.5 times when immersed in liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at $y=4$ cm.

15. (c) : Let W be the work function of metal. Then

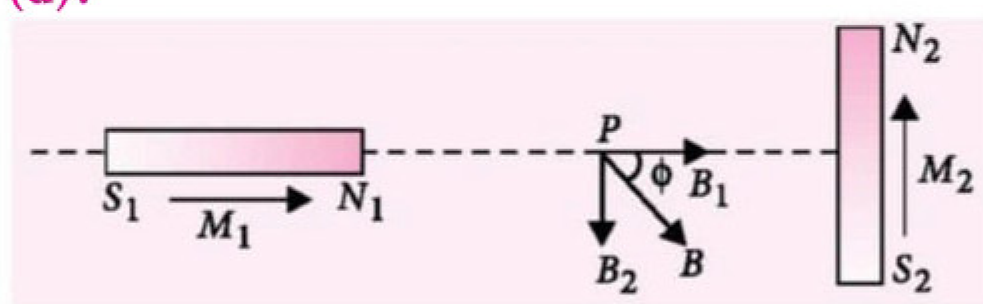
$$eV_0=\frac{hc}{330\times10^{-9}}-W \quad ...(i)$$

$$e(2V_0)=\frac{hc}{220\times10^{-9}}-W \quad ...(ii)$$

Solving these two equations, we get

$$V_0=\frac{10^9\times h\times c}{110\times e\times6}=\frac{10^9\times6.6\times10^{-34}\times3\times10^8}{110\times1.6\times10^{-19}\times6}=\frac{15}{8}\text{ volt}$$

16. (d):



Let point P be the midpoint between the dipoles. The point P will be in end-on position with respect to one dipole and in broad-side on position with respect to the other.

$$\therefore\quad B_1=\frac{\mu_0}{4\pi}\frac{2M_1}{r_1^3}=\frac{10^{-7}\times2\times2}{(1)^3}=4\times10^{-7}\text{ T}$$

and $$B_2=\frac{\mu_0}{4\pi}\frac{M_2}{r_2^3}=\frac{10^{-7}\times2}{(1)^3}=2\times10^{-7}\text{ T}$$

As B_1 and B_2 are perpendicular to each other, therefore the resultant magnetic field at point P is

$B=\sqrt{B_1^2+B_2^2}=\sqrt{(4\times10^{-7})^2+(2\times10^{-7})^2}$

$=10^{-7}\sqrt{16+4}=10^{-7}\sqrt{20}=2\sqrt{5}\times10^{-7}\text{ T}$

17. (b): As a convex lens alone can form a real image as well as a virtual image, therefore, the lens in the present question is a convex lens. Let, f be the focal length of the lens and m be the magnification produced.

In the first case, when image is real,

$u=-16$ cm, $v=(m\times16)$ cm

As $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\therefore\quad \frac{1}{16m}+\frac{1}{16}=\frac{1}{f}\text{ or }1+\frac{1}{m}=\frac{16}{f} \quad ...(i)$$

In the second case, when image is virtual.

$u=-6$ cm, $v=(-6m)$ cm

From $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-6m}+\frac{1}{6}=\frac{1}{f}\text{ or }1-\frac{1}{m}=\frac{6}{f} \quad ...(ii)$$

Adding eqn (i) and eqn (ii), we have

$2=\frac{22}{f}$ or $f=\frac{22}{2}=11\,\text{cm}$

18. (c) : When drop is stationary, then

q_1E = Weight – Upthrust

$q_1E = 6\pi\eta r v_0$ or $q_1 = 6\pi\eta r v_0 / E$

When drop moves upwards, then

$$3q = \frac{6\pi\eta r(v_0 + v_0)}{E} = 2\times\left(\frac{6\pi\eta r v_0}{E}\right) = 2q_1$$

$\therefore \quad q_1 = \frac{3}{2}q$

19. (c)

20. (d): As per question, time of flight for both the balls is equal.

$\frac{2u_1}{g} = \frac{2u_2\cos\theta}{g}$ or $u_1 = u_2\cos\theta$ ...(i)

For first ball; $\frac{1}{2}mu_1^2 = mgh_1$ ...(ii)

For second ball; $\frac{1}{2}2m(u_2\cos\theta)^2 = 2mgh_2$ ...(iii)

or $\frac{1}{2}mu_1^2 = mgh_2$ [From (i)]

$\therefore \quad h_1 = h_2$

21. (c) : Here, $m_1 = 0.5$ kg, $u_1 = 2.0$ m s^{-1}

$m_2 = 1.0$ kg, $u_2 = 0$

If v is velocity of the system after collision, then according to the principle of conservation of linear momentum,

$(m_1 + m_2)\,v = m_1 u_1 + m_2 u_2 = m_1 u_1$

$$v = \frac{m_1 u_1}{m_1 + m_2} = \frac{0.5\times 2.0}{0.5 + 1.0}$$

$$v = \frac{1.0}{1.5} = \frac{2}{3}\text{ m s}^{-1}$$

Energy loss = initial energy – final energy

$$= \frac{1}{2}m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}\times 0.5\times(2.0)^2 - \frac{1}{2}(0.5 + 1.0)\left(\frac{2}{3}\right)^2$$

$$= 1.0 - \frac{1.5}{2}\times\frac{4}{9} = 0.67\text{ J}$$

22. (c) : Let $m_1 = m_2 = m_3 = m$

Let s_1, s_2, s_3 be the respective specific heats of the three liquids.

(i) When A and B are mixed,

temperature of mixture = 16° C

As heat gained by A = heat lost by B

$\therefore \quad ms_1(16 - 12) = ms_2(19 - 16)$

$4s_1 = 3s_2$ ...(i)

(ii) When B and C are mixed,

temperature of mixture = 23°C

As heat gained by B = heat lost by C,

$ms_2(23 - 19) = ms_3(28 - 23)$

$\therefore \quad 4s_2 = 5s_3$ ...(ii)

From eqns (i) and (ii), $s_1 = \frac{3}{4}s_2 = \frac{15}{16}s_3$

When A and C are mixed, suppose temperature of mixture = t

heat gained A = heat lost by C

$ms_1(t - 12) = ms_3(28 - t)$

$\frac{15}{16}s_3(t - 12) = s_3(28 - t)$

$15t - 180 = 448 - 16t$

$31t = 448 + 180 = 628 \qquad \therefore\ t = 20.2°\text{C}$

23. (b): As $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

or $\frac{13}{6}R = \frac{1C_{V_1} + 2C_{V_2}}{1+2}$ or $C_{V_1} + 2C_{V_2} = \frac{13}{2}R$

If first gas is monoatomic, then $C_{V_1} = \frac{3}{2}R$ and second gas is diatomic, then $C_{V_2} = \frac{5}{2}R$.

$$\therefore \quad C_{V_1} + 2C_{V_2} = \frac{3}{2}R + 2\times\frac{5}{2}R = \frac{13}{2}R$$

It means the option (b) is true.

24. (c) : Let k be the force constant of smaller piece of spring. Then the longer piece is a combination of three smaller pieces in series. Their effective force constant $k_1 = k/3$. In a bigger spring, the smaller pieces are connected in series, so

$\frac{1}{K} = \frac{1}{k} + \frac{3}{k} = \frac{4}{k}$ or $k = 4K$

$\therefore \quad k_1 = \frac{4K}{3}$

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \text{ and } \upsilon' = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = \frac{1}{2\pi}\sqrt{\frac{4K/3}{m}} = \frac{2\upsilon}{\sqrt{3}}$$

25. (c) : Velocity of longitudinal waves, $v_1 = \sqrt{\frac{Y}{\rho}}$

velocity of transverse waves, $v_2 = \sqrt{\frac{T}{m}}$

If a is area of cross-section of string

then $m = \frac{\text{mass}}{\text{length}} = \frac{\text{mass}}{\text{volume}}\times\text{area} = \rho a$

$\therefore\ v_2 = \sqrt{\frac{T}{\rho a}},\ \frac{v_1}{v_2} = \sqrt{\frac{Y}{\rho}\cdot\frac{\rho a}{T}} = \sqrt{\frac{Ya}{T}}$

As $Y = \frac{F}{a\Delta l/l} = \frac{T}{a(\Delta l/l)}$

$\therefore\ \frac{v_1}{v_2} = \sqrt{\frac{T}{a\left(\frac{\Delta l}{l}\right)}\frac{a}{T}} = \left(\frac{\Delta l}{l}\right)^{-1/2}$

we are given, $\frac{\Delta l}{l} = \frac{1}{n}$

$\therefore\ \frac{v_1}{v_2} = \left(\frac{1}{n}\right)^{-1/2} = \sqrt{n}$

If υ_1 and υ_2 are the fundamental frequencies of longitudinal and transverse waves then

$v_1 = \upsilon_1\lambda$ and $v_2 = \upsilon_2\lambda$. $\therefore\ \frac{\upsilon_1}{\upsilon_2} = \sqrt{n}$

26. (d): As the source is moving towards the hill (the listener), therefore, apparent frequency of horn striking the hill is

$$\upsilon' = \frac{v}{v - v_s}\times\upsilon = \frac{330\times600}{330-30} = 660\,\text{Hz}$$

For the reflected sound, driver acts as listener moving towards source (the hill)

$$\therefore\ \upsilon'' = \frac{(v+v_L)\upsilon}{v} = \frac{(330+30)660}{330} = 720\,\text{Hz}$$

27. (d): In forward biasing, both potential barrier V_B and the width of charge depleted region x decreases.

28. (a): Rate of change of radioactive nuclei,

$$\left(-\frac{dN}{dt}\right) = \lambda N$$

$$\because\ \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1620\times365\times24\times60\times60}$$

and $N = \frac{6.023\times10^{23}}{226}$

$$\therefore\left(-\frac{dN}{dt}\right) = \frac{0.693\times6.023\times10^{23}}{1620\times365\times24\times60\times60\times226}$$

$= 3.61\times10^{10}$ Bq

29. (a): $X_C > X_L$. Hence, current will lead the voltage.

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 10\sqrt{2}\,\Omega$$

$$\therefore\ I_{rms} = \frac{V_{rms}}{Z} = \frac{400/\sqrt{2}}{10\sqrt{2}} = 20\,\text{A}$$

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

$V_R = I_{rms}R = (20)(10) = 200$ V

30. (c): $V_{actual} = \left(\frac{100}{110}\right)V = V_1 = \frac{10}{11}V$

Equivalent resistance of 100 Ω and 900 Ω is 90 Ω

$$V_{measured} = \left(\frac{90}{90+10}\right)V = \frac{9}{10}V = V_2$$

$$\%\ \text{error} = \frac{V_1 - V_2}{V_1}\times100$$

$$= \frac{\left(\frac{10}{11}V\right) - \left(\frac{9}{10}\right)V}{\left(\frac{10}{11}\right)V}\times100 = 1\%$$

31. (a): Let the magnetic field is $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

Applying $\vec{F}_m = q(\vec{v}\times\vec{B})$ we have,

$q[-\hat{j}+\hat{k}] = q[(\hat{i})\times(B_1\hat{i}+B_2\hat{j}+B_3\hat{k})] = q[B_2\hat{k} - B_3\hat{j}]$

Comparing two sides, we get,

$B_2 = 1$ and $B_3 = 1$

Further, $q[\hat{i}-\hat{k}] = q[(\hat{j})\times(B_1\hat{i}+B_2\hat{j}+B_3\hat{k})]$

$= q[-B_1\hat{k} + B_3\hat{i}]$

Again comparing we get,

$B_1 = 1$ and $B_3 = 1$

$\therefore\ B = (\hat{i}+\hat{j}+\hat{k})\,\text{Wb m}^{-2}$

32. (b): In the given system, $a = \frac{(m_1 - m_2)g}{m_1+m_2} = \frac{g}{8}$

$\therefore\ \frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{8}$

$8m_1 - 8m_2 = m_1 + m_2$

$7m_1 = 9m_2$ or $\frac{m_1}{m_2} = \frac{9}{7}$

33. (a): Tension is maximum at the lowest point and minimum at the highest point.

Tension at the lowest point,

$$T_L = mg + \frac{mv_L^2}{r} = mg + \frac{7mgr}{r} = 8mg \quad \left(\because\ v_L = \sqrt{7gr}\right)$$

Tension at the highest point,

$$T_H = \frac{mv_H^2}{r} - mg = \frac{m(v_L^2 - 4gr)}{r} - mg$$

$$= \frac{m(7gr - 4gr)}{r} - mg \quad \left(\because\ v_L^2 - v_H^2 = 4gr\right)$$

$= 3mg - mg = 2mg$

$\therefore\ \frac{T_L}{T_H} = \frac{4}{1}$ or $\frac{T_{max}}{T_{min}} = \frac{4}{1}$

34. (c) : Moment of inertia of coin of mass m and radius r about the axis passing through the centre of mass and perpendicular to its plane is

$$I = \frac{mr^2}{2} \quad \text{...(i)}$$

Moment of inertia of disc of mass m and radius $2r$ about the axis passing through the centre of mass and perpendicular to its plane is

$$I' = \frac{m(2r)^2}{2} = 4I \quad \text{(Using (i))}$$

35. (a) : Pressure outside the bigger drop = P_1
Pressure inside the bigger drop = P_2
Radius of bigger drop, r_1 = 3 cm

Excess pressure $= P_2 - P_1 = \frac{4S}{r_1} = \frac{4S}{3}$

Pressure inside small drop = P_3

Excess pressure $= P_3 - P_2 = \frac{4S}{r_2} = \frac{4S}{1}$

Pressure difference between inner side of small drop and outer side of bigger drop

$$= P_3 - P_1 = \frac{4S}{3} + \frac{4S}{1} = \frac{16S}{3}$$

This pressure difference should exist in a single drop of radius r.

$$\therefore \quad \frac{4S}{r} = \frac{16S}{3} \quad \text{or} \quad r = \frac{3}{4}\text{ cm} = 0.75\text{ cm}$$

36. (a) : Efficiency of the heat engine,

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

where T_1 and T_2 are the temperatures of source and sink respectively.
When T_1 and T_2 both are decreased by 100 K each, $(T_1 - T_2)$ stays constant. T_1 decreases.
$\therefore$ η increases.

37. (a) : For equilibrium of $(m_1 + m_2)$, $x_1 = \frac{(m_1+m_2)g}{k}$

and for equilibrium of m_2, $x_2 = \frac{m_2 g}{k}$

$\therefore$ Amplitude of oscillation will be

$$A = x_1 - x_2 = \frac{(m_1+m_2)g}{k} - \frac{m_2 g}{k} = \frac{m_1 g}{k}$$

38. (d): In case of diffraction at a single slit, the position of minima is given by

$$d\sin\theta = n\lambda$$

If θ is small, $\sin\theta = \theta = \frac{y}{D}$

So, the position of first minimum relative to centre will be given by

$d(y/D) = \lambda$, *i.e.*, $y = (D/d)\lambda$

Here, D = 2 m; $d = 1 \times 10^{-3}$ m and $\lambda = 6 \times 10^{-7}$ m

So, $y = \frac{2 \times 6 \times 10^{-7}}{1 \times 10^{-3}} = 1.2$ mm

$\therefore$ Distance between first minima on either side of central maxima, $\Delta y = 2y = 2.4$ mm.

39. (b): At every instant, the ratio of the magnitude of the electric field to that of the magnetic field in an electromagnetic wave equals the speed of light.

40. (b): According to Einstein's photoelectric equation
$eV_s = h\upsilon - h\upsilon_0$
where, υ = Incident frequency
υ_0 = Threshold frequency
V_s = Cut-off or stopping potential

or $V_s = \frac{h}{e}(\upsilon - \upsilon_0)$

Substituting the given values, we get

$$V_s = \frac{6.63\times10^{-34}(8.2\times10^{14} - 3.3\times10^{14})}{1.6\times10^{-19}} \approx 2\text{ V}$$

41. (c) : Three half lives of A are equivalent to six half lives of B. As number of nuclei left are equal in the two cases.

Therefore, $N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$

$$\frac{N_A}{N_B} = \frac{(1/2)^6}{(1/2)^3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

42. (c) : To have the same wavelength of some spectral lines from different hydrogen like atoms, one must have

$$\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = Z^2\left[\frac{1}{p_f^2} - \frac{1}{p_i^2}\right]$$

where n_f and n_i are principal quantum numbers of final and initial orbits for hydrogen atom and p_f and p_i are those for He^+ ion. Also, Z = 2. This gives $p_f = 2n_f$ and $p_i = 2n_i$.

43. (c)

44. (c) : At resonance frequency, current is maximum.

45. (b) : Electric field on the surface of a conducting sphere is $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

$$\therefore \quad q = Er^2 \cdot 4\pi\varepsilon_0$$

$$= \frac{3\times10^6 \times (2.5)^2}{9\times10^9} = 2.08\times10^{-3}\text{C}$$

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PRACTICE PAPER

Exam on : 1^{st} to 6^{th} September 2020

JEE MAIN 2020

1. A body is projected vertically upwards from the surface of the earth with a velocity sufficient to carry it to infinity. If R is the radius of the earth, the time taken by the body to reach to a height h from ground is

(a) $\sqrt{\frac{2R}{g}}$ (b) $\frac{1}{3}\sqrt{\frac{2R}{g}}$

(c) $\frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(1+\frac{h}{R}\right)^{3/2}\right]$

(d) $\frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(1+\frac{h}{R}\right)^{3/2}-1\right]$.

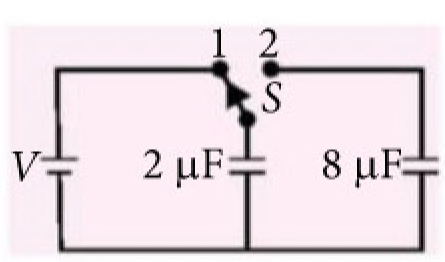
2. A cannon of mass $2m$ located at the base of an inclined plane shoots a shell of mass m in horizontal direction with velocity v_0. The angle of inclination of the plane is 45° and the coefficient of friction between the cannon and the plane is 0.5. The height to which the cannon ascends the plane as a result of recoil is

(a) $\frac{v_0^2}{2g}$ (b) $\frac{v_0^2}{6g}$ (c) $\frac{v_0^2}{g}$ (d) $\frac{v_0^2}{12g}$.

3. An α-particle and a proton are fired through the same magnetic field which is perpendicular to their velocity vectors. The α-particle and the proton move such that radius of curvature of their paths is same. Find the ratio of their de Broglie wavelengths.

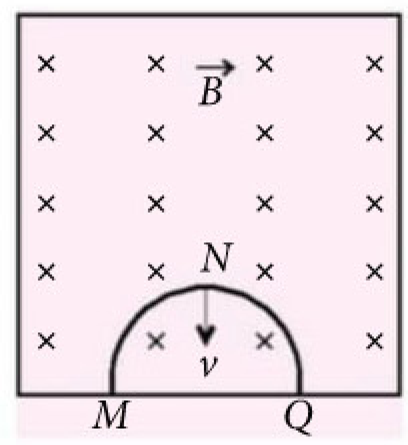
(a) 2 : 3 (b) 3 : 4 (c) 5 : 7 (d) 1 : 2

4. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(a) 10% (b) 20% (c) 75% (d) 80%.

5. A thin semicircular conducting ring of radius R is falling with its plane vertical in horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v, the potential difference developed across the ring is



(a) Zero

(b) $Bv\pi R^2/2$ and M is at higher potential

(c) πRBv and Q is at higher potential

(d) $2RBv$ and Q is at higher potential.

6. If 10% of a radioactive substance decays in every 5 years, then the percentage of the substance that will have decayed in 20 years will be

(a) 40% (b) 50% (c) 65.6% (d) 34.4%.

7. An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency $\frac{25}{\pi}$ Hz. At the position x = 0.04 m, the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is (potential energy is zero at mean position)

(a) 6 cm (b) 4 cm (c) 8 cm (d) 2 cm.

8. A car starting from rest, accelerates at the rate a through a distance s, then continues at constant speed for time t and then decelerates at the rate $a/2$ to come to rest. If the total distance travelled is $15s$, then

(a) $s = at$ (b) $s=\frac{1}{6}at^2$

(c) $s=\frac{1}{2}at^2$ (d) None of these.

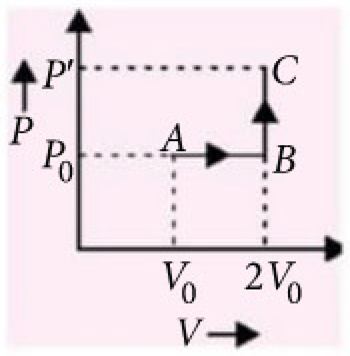
9. Two conductors have the same resistance at 0 °C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients

of their series and parallel combinations are nearly

(a) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (b) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (d) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$.

10. The pressure P' of one mole of monatomic gas in the process ABC, shown in the diagram, if $\Delta U/\Delta Q = 6/7$ is



(a) $\frac{5}{2}P_0$ (b) $\frac{3}{2}P_0$

(c) $2P_0$ (d) $\frac{7}{2}P_0$

11. A thin uniform rod AB of mass m and length L is hinged at one end A to the level floor. Initially it stands vertically and is allowed to fall freely to the floor in the vertical plane. The angular velocity of the rod, when its end B strikes the floor is (g is acceleration due to gravity)

(a) $\left(\frac{mg}{L}\right)$ (b) $\left(\frac{mg}{3L}\right)^{1/2}$

(c) $\left(\frac{g}{L}\right)$ (d) $\left(\frac{3g}{L}\right)^{1/2}$.

12. A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air on the upper and lower surfaces of the wings are 270 km h^{-1} and 234 km h^{-1} respectively, then the mass of the plane is
(Take the density of the air = 1 kg m^{-3})

(a) 1550 kg (b) 1750 kg
(c) 3500 kg (d) 3200 kg.

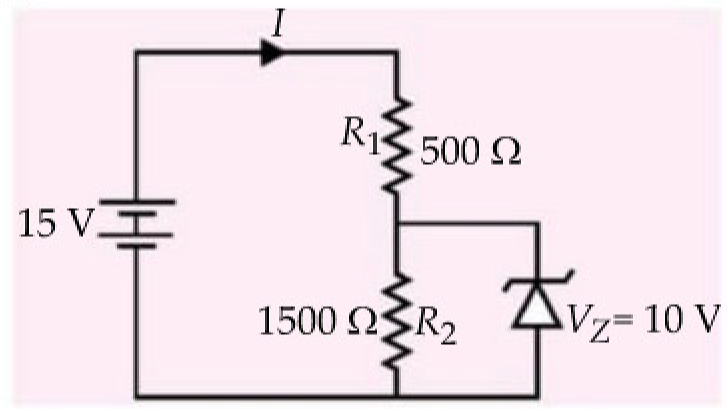
13. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is

(a) $Q/2$ (b) $Q/\sqrt{3}$ (c) $Q/\sqrt{2}$ (d) Q.

14. A parachutist drops freely from an aeroplane for 10 s and then the parachute opens out. Then he descends with a net retardation of 12 m s^{-2}. If he strikes the ground with a velocity of 20 m s^{-1}, then the height at which he bails out of the plane is (g = 10 m s^{-2})

(a) 400 m (b) 500 m (c) 800 m (d) 900 m.

15. In the circuit given, the current through the Zener diode is



(a) 10 mA (b) 6.67 mA
(c) 5 mA (d) 3.33 mA.

16. The minimum kinetic energy required for ionization of a hydrogen atom is E_1 in case electron is collided with hydrogen atom. It is E_2 if the hydrogen ion is collided and E_3 when helium ion is collided. Then,

(a) $E_1 = E_2 = E_3$ (b) $E_1 > E_2 > E_3$
(c) $E_1 < E_2 < E_3$ (d) $E_1 > E_3 > E_2$.

17. The apparent coefficient of expansion of a liquid when heated, filled in vessel A and B of identical volumes, is found to be γ_1 and γ_2 respectively. If α_1 be the linear expansion of A then that of B will be

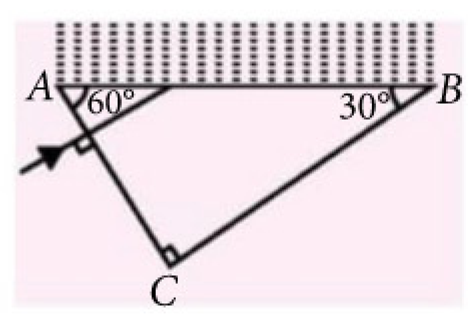
(a) $\frac{(\gamma_1-\gamma_2)}{3}-\alpha_1$ (b) $\frac{(\gamma_2-\gamma_1)}{3}+\alpha_1$

(c) $\frac{(\gamma_2-\gamma_1)}{3}-\alpha_1$ (d) $\frac{(\gamma_1-\gamma_2)}{3}+\alpha_1$.

18. An electromagnetic wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure P on it. Which of the following statement is not true?

(a) Radiation pressure is I/c if the wave is totally absorbed.
(b) Radiation pressure is I/c if the wave is totally reflected.
(c) Radiation pressure is $2I/c$ if the wave is totally reflected.
(d) Radiation pressure is in the range $I/c < P < 2I/c$ for real surfaces.

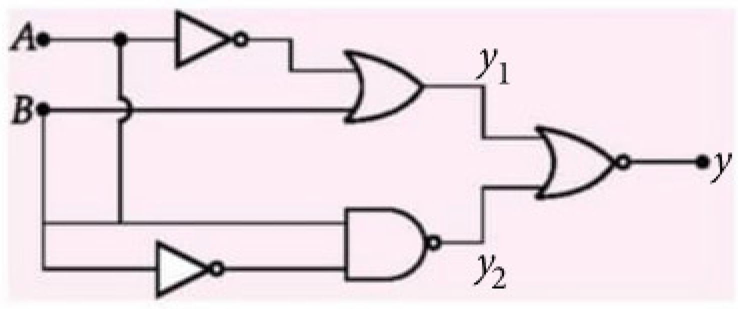
19. ACB is right-angled prism with other angles as 60° and 30°. Refractive index of the prism is 1.5. AB has thin layer of liquid on it as shown in the figure.



Light falls normally on the face AC. For total internal reflections, maximum refractive index of the liquid is

(a) 1.4 (b) 1.3 (c) 1.2 (d) 1.6.

20. Figure shows a system of logic gates for $y = 1$. Which one of the following options is satisfied?



(a) $A = 0, B = 1$ (b) $A = 1, B = 1$
(c) $A = 1, B = 0$ (d) $A = 0, B = 0$.

NUMERICAL VALUE TYPE

21. The measured mass and volume of a body are 22.42 g and 4.7 cm^3 respectively with possible errors 0.01 g and 0.1 cm^3. The maximum error in density is about ________%

22. A wire of density 9 g cm^{-3} is stretched between two clamps 1.00 m apart while subjected to an extension of 0.05 cm. The lowest frequency (in Hz) of transverse vibrations in the wire is ______ (Assume Young's modulus, $Y = 9 \times 10^{10}$ N m^{-2})

23. A TV transmission tower antenna is at a height of 20 m. The percentage increase in area covered in case if the receiving antenna is at ground level to that at a height of 25 m approximately is ________% (Radius of earth = 6.4×10^6 m)

24. The index of refraction of a glass plate is 1.48 at θ_1 = 30 °C and varies linearly with temperature with a coefficient of 2.5×10^{-5} °C^{-1}. The coefficient of linear expansion of the glass is 0.5×10^{-5} °C^{-1}. At 30 °C, the length of the glass plate is 3 cm. This plate is placed along the length in front of one of the slits in Young's double-slit experiment. If the plate is being heated so that its temperature increases at a rate of 5 °C min^{-1}, the light source has wavelength λ = 589 nm and the glass plate initially is at θ = 30 °C. The number of fringes that shift on the screen in each minute is nearly ______ (use approximation)

25. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be ________ $\times 10^{-2}$ s.

SOLUTIONS

1. (d) : Let v be the velocity of the projected body at distance r from the centre of earth. Using the law of conservation of mechanical energy, we have

$$\frac{1}{2}mv^2 + \left(\frac{-GMm}{r}\right) = \frac{1}{2}mv_e^2 + \left(\frac{-GMm}{R}\right)$$

or $$v^2 = v_e^2 + \frac{2GM}{R}\left[\frac{R}{r} - 1\right]$$

As $v_e = \sqrt{2gR}$ and $g = (GM/R^2)$

so $v^2 = 2gR + 2gR[(R/r) - 1]$

or $v = \sqrt{2gR^2/r}$

$$\therefore \frac{dr}{dt} = \frac{R\sqrt{2g}}{\sqrt{r}} \text{ or } \int_0^t dt = \frac{1}{R\sqrt{2g}}\int_R^{R+h}\sqrt{r}\,dr$$

or $$t = \frac{2}{3}\frac{1}{R\sqrt{2g}}[(R+h)^{3/2} - R^{3/2}]$$

$$\therefore t = \frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(1+\frac{h}{R}\right)^{3/2} - 1\right]$$

2. (d) : From conservation of linear momentum, recoil velocity of cannon,

$$v = \frac{mv_0}{2m} = \frac{v_0}{2}$$

From conservation of mechanical energy,
KE of cannon = increase in gravitational PE + work done against friction

$$\frac{1}{2}(2m)v^2 = (2m)gh + \mu(2m)g\cos 45° \times \frac{h}{\sin 45°}$$

$$m\left(\frac{v_0}{2}\right)^2 = 3mgh \therefore h = \frac{v_0^2}{12g}$$

3. (d) : Magnetic force experienced by a charged particle in a magnetic field is given by

$$F_B = |q\vec{v} \times \vec{B}| = qvB\sin\theta$$

or $F_B = qvB$ [as $\theta = 90°$]

Hence, $Bqv = \frac{mv^2}{r}$ or $mv = qBr$

The de Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{qBr}$$

Required ratio, $$\frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{Proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

$\because \quad \frac{r_\alpha}{r_p} = 1$ and $\frac{q_\alpha}{q_p} = 2$

$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$

4. (d) : $U_i = \frac{1}{2}CV^2 = \frac{1}{2}(2)V^2 = V^2, q_i = CV = 2V$

On turning the switch S to position 2, if q is the charge on 8 μF capacitor, charge on 2 μF capacitor is $(2V - q)$.

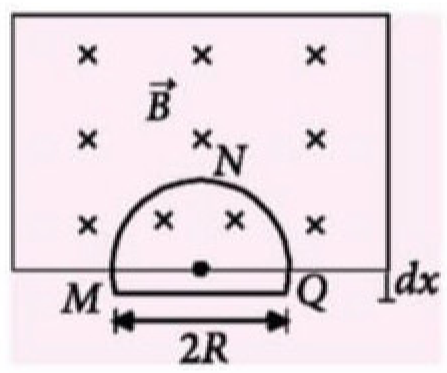
Thus, $\frac{2V-q}{2} = \frac{q}{8}$, whence $q = \frac{8V}{5}$ $\therefore \quad (2V-q) = \frac{2V}{5}$

Energy dissipated,

$$\Delta U = U_i - U_f = V^2 - \left[\frac{(8V/5)^2}{2\times 8} + \frac{(2V/5)^2}{2\times 2}\right]$$

$$\Delta U = \frac{4V^2}{5} = \frac{4}{5}(V^2) = \frac{4}{5}(U_i) = 80\% \text{ of } U_i$$

5. (d) : At the position MNQ, the ring is at the point of moving out of the magnetic field. Its speed of fall is v. Due to fall, the flux associated with the ring changes. An emf is therefore induced in the ring.



Induced emf $= \frac{-d\phi}{dt}$ or $\varepsilon = \frac{-d}{dt}(BA\cos\theta)$

or $\varepsilon = -B\frac{dA}{dt}\cos 0°$; the ring falls with its plane vertical in horizontal magnetic induction $\vec{B}$.

or $\varepsilon = -B.\frac{d}{dt}(2R\,dx)$ or $\varepsilon = -2RB\left(\frac{dx}{dt}\right)$

$|\varepsilon| = 2RBv$

6. (d)

7. (a) : Angular velocity, $\omega = 2\pi\upsilon = \sqrt{\frac{k}{m}}$

$\therefore \quad k = (2\pi\upsilon)^2 m$

Total energy of oscillation is 0.5 + 0.4 = 0.9 J

$\therefore \quad 0.9 = \frac{1}{2}kA^2$

$$A = \sqrt{\frac{1.8}{k}} = \sqrt{\frac{1.8}{(2\pi\upsilon)^2 m}}$$

$$= \frac{1}{2\pi\upsilon}\sqrt{\frac{1.8}{0.2}} = \frac{1}{2\pi\left(\frac{25}{\pi}\right)}\sqrt{\frac{1.8}{0.2}} = \frac{3}{50}\text{ m} = 6\text{ cm}$$

8. (d) **9. (a)**

10. (a) : In process AB, pressure = constant.

$\therefore \quad V \propto T$

$V_B = 2V_0$ so, $T_B = 2T_0$ (if $T_A = T_0$)

$\Delta W = P_0(2V_0 - V_0) = P_0V_0 = RT_0$

and $\Delta Q = C_P\Delta T = \frac{5}{2}RT_0$

$\Delta U = \Delta Q - \Delta W = \frac{3}{2}RT_0$

In process BC, volume = constant.

$\therefore \quad P \propto T$

$P_B = P_0$ but $T_B = 2T_0$

So, $P' = KP_0$, $T_C = KT_B = K(2T_0)$

$\Delta W = 0$ and $\Delta Q = \Delta U = C_V\Delta T$

$= \frac{3}{2}R \times 2T_0(K-1) = 3RT_0(K-1)$

$\Delta Q_{\text{Total}} = \frac{5}{2}RT_0 + 3RT_0(K-1)$ ...(i)

$\Delta U_{\text{Total}} = \frac{3}{2}RT_0 + 3RT_0(K-1)$ ...(ii)

Given, $\Delta U/\Delta Q = 6/7$ ...(iii)

Comparing eqns. (i), (ii) and (iii), $K = 5/2$

$\therefore \quad P' = \frac{5}{2}P_0$

11. (d) : As the rod is hinged at one end, its moment of inertia about this end is $I = \frac{mL^2}{3}$

Total energy in upright position = total energy on striking the floor

$$0 + \frac{mgL}{2} = \frac{1}{2}I\omega^2 + 0 = \frac{1}{2}\frac{mL^2}{3}\omega^2$$

$$g = \frac{L\omega^2}{3} \text{ or } \omega = \sqrt{\frac{3g}{L}}$$

12. (c)

13. (c): Q denotes maximum charge on capacitor.

Let q denotes charge when energy is equally shared

$\therefore \quad \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{q^2}{C}$ or $Q^2 = 2q^2$

$\therefore \quad q = Q/\sqrt{2}$

14. (d) : Velocity acquired after 10 s of fall,

$u = gt = 10 \times 10 = 100 \text{ m s}^{-1}$

Distance travelled, $s_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 10^2 = 500$ m

Velocity on reaching the ground, $v = 20\ \text{m s}^{-1}$
We have, $u = 100\ \text{m s}^{-1}$, $v = 20\ \text{m s}^{-1}$, $a = -12\ \text{m s}^{-2}$ and $s = s_2$ (say).
Using, $v^2 = u^2 + 2as$,
$20^2 = 100^2 + 2(-12)s_2$ or $s_2 = 400$ m
Total height at which he comes out of the plane
$= s_1 + s_2 = 500 + 400 = 900$ m

15. (d)

16. (b) : Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv_0 - 0 = (m + m_H)u$$

where m is the mass of incident particle, m_H the mass of hydrogen atom, v_0 the initial velocity of incident particle, and u the final common velocity of the particle after collision. Prior to collision, the KE of the incident particle was $E_0 = \dfrac{mv_0^2}{2}$

The total kinetic energy after collision

$$E = \frac{(m+m_H)u^2}{2} = \frac{m^2 v_0^2}{2(m+m_H)}$$

The decrease in kinetic energy must be equal to ionization energy. Therefore,

$$E' = E_0 - E = \left(\frac{m_H}{m+m_H}\right)E_0$$

i.e., $\dfrac{E'}{E_0} = \dfrac{1}{1+\dfrac{m}{m_H}}$

i.e., the greater the mass m, the smaller the fraction of initial kinetic energy that be used for ionization.

17. (b) : Let V be the volume of the liquid and ΔT the rise in temperature.
Since, apparent expansion = true expansion – expansion of vessel

$\therefore\quad V\gamma_1\Delta T = V\gamma\Delta T + V(3\alpha_1)\Delta T$

or $\quad \gamma_1 = \gamma + 3\alpha_1$ ...(i) (for vessel A)

and $\quad \gamma_2 = \gamma + 3\alpha_2$ ...(ii) (for vessel B)

where γ is the coefficient of real expansion of the liquid.
Subtracting eqn. (i) from eqn. (ii),

$\gamma_2 - \gamma_1 = 3(\alpha_2 - \alpha_1)$

or $\alpha_2 - \alpha_1 = (\gamma_2 - \gamma_1)/3$ or $\alpha_2 = [(\gamma_2 - \gamma_1)/3] + \alpha_1$

18. (b) : Momentum per unit time per unit area

$$= \frac{\text{intensity}}{\text{speed of wave}} = \frac{I}{c}$$

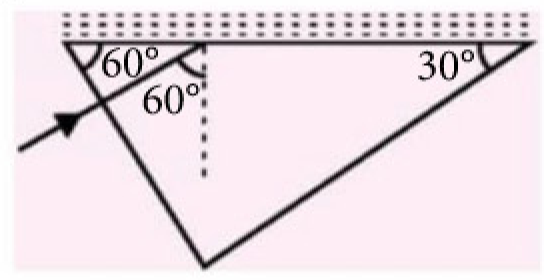
Change in momentum per unit time per unit area $= \Delta I/c$ = radiation pressure (P), *i.e.*, $P = \Delta I/c$

Momentum of incident wave per unit time per unit area = I/c
When wave is fully absorbed by the surface, the momentum of the reflected wave per unit time per unit area = 0
Radiation pressure (P) = change in momentum per unit time per unit area $= \dfrac{\Delta I}{c} = \dfrac{I}{c} - 0 = \dfrac{I}{c}$
When wave is totally reflected, then momentum of the reflected wave per unit time per unit area = $-I/c$

Radiation pressure $(P) = \dfrac{I}{c} - \left(-\dfrac{I}{c}\right) = \dfrac{2I}{c}$

Here, P lies between $\dfrac{I}{c}$ and $\dfrac{2I}{c}$.

19. (b) : Clearly, $i_c \le 60°$
So, maximum possible value of i_c is 60°.



Now, $\ {}^{l}\mu_g = \dfrac{1}{\sin i_c}$

$\dfrac{\mu_g}{\mu_l} = \dfrac{1}{\sin i_c}$

or $\quad \mu_l = \mu_g \sin i_c = 1.5 \sin 60° = 1.5 \times \dfrac{\sqrt{3}}{2}$
$= 1.5\ (0.866) = 1.299 = 1.3$

20. (c) : Using Boolean algebra for the gates, we have

$$y_1 = \overline{A} + B;\ y_2 = \overline{A\cdot\overline{B}} = \overline{A} + \overline{\overline{B}} = \overline{A} + B$$

$$y = \overline{y_1 + y_2}$$

Using $A = 1, B = 0, y_1 = \overline{1} + 0 = 0$ and $y_2 = \overline{1} + 0 = 0$

$\therefore\quad y = \overline{0+0} = 1$

Thus option (c) is true.

21. (2) : As $\rho = \dfrac{M}{V}$

$$\therefore\quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{0.01}{22.42} + \frac{0.1}{4.7} = \frac{0.047 + 2.242}{22.42 \times 4.7}$$

Monthly Test Drive CLASS XII ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(c)
6.	(a)	**7.**	(d)	**8.**	(c)	**9.**	(b)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(c)	**14.**	(a)	**15.**	(b)
16.	(a)	**17.**	(c)	**18.**	(a)	**19.**	(d)	**20.**	(a, b, d)
21.	(b, d)	**22.**	(b, c)	**23.**	(a,d)	**24.**	(2)	**25.**	(4)
26.	(2)	**27.**	(c)	**28.**	(b)	**29 .**	(a)	**30.**	(a)

$$\frac{\Delta\rho}{\rho}\times 100=\frac{2.289\times 100}{22.42\times 4.7}\%=2.172\%\approx 2\%$$

22. (35) : Tension in the wire, $T=\frac{YA\Delta l}{l}$

Lowest frequency $=\frac{v}{2l}=\frac{\sqrt{T/\rho A}}{2l}$

$$\upsilon=\frac{1}{2l}\sqrt{\frac{YA\Delta l}{\rho lA}}=\frac{1}{2l}\sqrt{\frac{Y\Delta l}{\rho l}}$$

Substituting the values,

$$\upsilon=\frac{1}{2\times 1}\sqrt{\frac{9\times 10^{10}\times 0.05\times 10^{-2}}{(9000)(1)}}\approx 35\text{ Hz}$$

23. (349) : Here, $h_T=20$ m, $R=6.4\times 10^6$ m $=64\times 10^5$ m

If the receiving antenna is at ground level,

Range, $d=\sqrt{2h_T R}=\sqrt{2\times 20\times(64\times 10^5)}$

$= 16\times 10^3$ m $= 16$ km

Area covered, $A=\pi d^2=\frac{22}{7}\times(16)^2=804.6\text{ km}^2$

If the receiving antenna is at a height of 25 m,

Range, $d_1=\sqrt{2h_T R}+\sqrt{2h_R R}$

$=\sqrt{2\times 20\times(64\times 10^5)}+\sqrt{2\times 25\times(64\times 10^5)}$

$= 16\times 10^3 + 17.9\times 10^3 = 33.9$ km

Area covered, $A_1=\pi d_1^2=\frac{22}{7}\times(33.9)^2=3611.8\text{ km}^2$

Percentage increase in area $=\frac{A_1-A}{A}\times 100$

$=\left(\frac{3611.8-804.6}{804.6}\right)\times 100\%$

$= 348.9\% \simeq 349\%$

24. (11) : Path difference $=\mu t-\mu_0 t_0=n\lambda_0$

where n is the number of fringes that shift on the screen.

$$\therefore\quad \frac{\mu_0(1+\alpha_1\Delta\theta)t_0(1+\alpha_2\Delta\theta)-\mu_0 t_0}{\lambda_0}=n$$

$$\frac{\mu_0 t_0(\alpha_1+\alpha_2)\Delta\theta}{\lambda_0}=n$$

($\because$ $\alpha_1\alpha_2(\Delta\theta)^2$ is very-very small.)

Given, $\mu_0 = 1.48$, $t_0 = 3\times 10^{-2}$ m

$\alpha_1 = 2.5\times 10^{-5}\,°C^{-1}$, $\alpha_2 = 0.5\times 10^{-5}\,°C^{-1}$, $\Delta\theta = 5\,°C$

and $\lambda_0 = 589$ nm

$$\therefore\quad n=\frac{1.48\times 3\times 10^{-2}(3\times 10^{-5})\times 5}{589\times 10^{-9}}=11$$

25. (66.6) : For a vibrating magnet, $T=2\pi\sqrt{\frac{I}{MB}}$

where $I = ml^2/12$, $M = xl$, x = pole strength of magnet

For three pieces together

$$I'=\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2\times\frac{3}{12}=\frac{ml^2}{9\times 12}=\frac{I}{9}$$

$$M'=(x)\left(\frac{l}{3}\right)\times 3=xl=M$$

$$\therefore\quad T'=2\pi\sqrt{\frac{I'}{M'B}}=2\pi\sqrt{\frac{I/9}{MB}}=\frac{1}{3}\times 2\pi\sqrt{\frac{I}{MB}}=\frac{T}{3}$$

So, $T'=\frac{2}{3}$ s $= 66.6\times 10^{-2}$ s

Exam on 27^{th} September

JEE 2020 PRACTICE PAPER ADVANCED

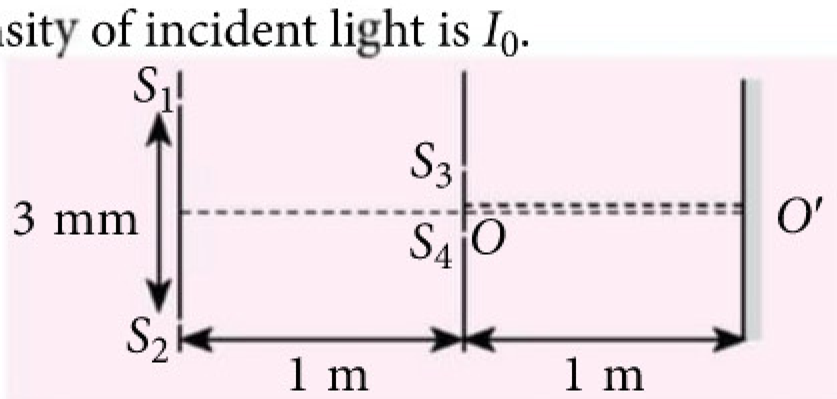
PAPER-I

SECTION 1 (Maximum Marks : 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. ONLY ONE of these four options is the correct answer.*
- *For each question, choose the correct option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +3 *If ONLY the correct option is chosen.*
Zero Marks : 0 *(If none of the options is chosen i.e. the question is unanswered).*
Negative Marks : –1 *In all other cases.*

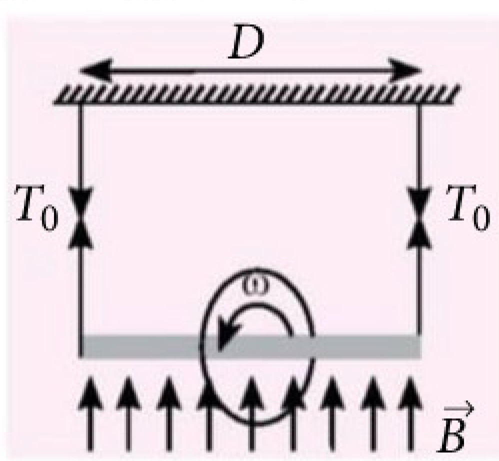
1. In the arrangement shown in figure, light of wavelength 6000 Å is incident on slits S_1 and S_2. Slits S_3 and S_4 have been opened such that S_3 is the position of first maximum above the central maximum and S_4 is the closest position, where intensity is same as that of the light used, below the central maximum. The point O is equidistant from S_1 and S_2 and O' is equidistant from S_3 and S_4. The intensity of incident light is I_0.



The intensity at O' (on the screen) and the intensity of the brightest fringe respectively be

(a) $3I_0$ and $9I_0$ (b) $9I_0$ and $3I_0$
(c) $2I_0$ and $4I_0$ (d) $4I_0$ and $2I_0$

2. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. The maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_0/2$ is



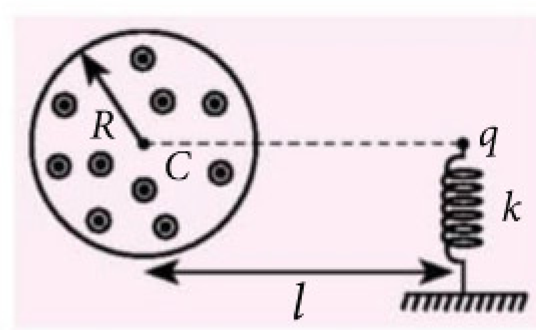
(a) $\dfrac{DT_0}{BQR}$ (b) $\dfrac{DT_0}{B^2QR}$ (c) $\dfrac{2DT_0}{QRB}$ (d) $\dfrac{DT_0}{QBR^2}$

3. From the surface of a wire of radius a carrying a direct current I, a positive charge q having mass m escapes with a velocity v_0 perpendicular to the surface. The maximum distance (x_{max}) of the electron from the axis of the wire before it turns back due to the action of the magnetic field generated by the current will be

(a) $x_{max} = ae^{\frac{2\pi m v_0}{q\mu_0 I}}$ (b) $x_{max} = ae^{-\frac{\pi m v_0}{q\mu_0 I}}$

(c) $x_{max} = ae^{\frac{m v_0}{qI}}$ (d) $x_{max} = ae^{\frac{\pi m v_0}{2q\mu_0 I}}$

4. There is a horizontal cylindrical uniform but time-varying magnetic field increasing at a constant rate dB/dt as shown in figure.



A charged particle having charge q and mass m is kept in equilibrium, at the top of a spring of spring constant k, in such a way that it is on the horizontal line passing through the center of the magnetic field as shown in the figure. The compression in the spring will be

(a) $\dfrac{1}{k}\left[mg - \dfrac{qR^2}{2l}\dfrac{dB}{dt}\right]$ (b) $\dfrac{1}{k}\left[mg + \dfrac{qR^2}{l}\dfrac{dB}{dt}\right]$

(c) $\dfrac{1}{k}\left[mg + \dfrac{2qR^2}{l}\dfrac{dB}{dt}\right]$ (d) $\dfrac{1}{k}\left[mg + \dfrac{qR^2}{2l}\dfrac{dB}{dt}\right]$

SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT (08) questions.*
- *Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

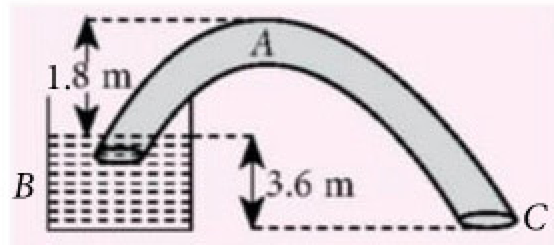
Full Marks :	*+4*	*If only (all) the correct option(s) is (are) chosen.*
Partial Marks :	*+3*	*If all the four options are correct but ONLY three options are chosen.*
Partial Marks :	*+2*	*If three or more options are correct but ONLY two options are chosen and both of which are correct.*
Partial Marks :	*+1*	*If two or more options are correct but ONLY one option is chosen and it is a correct option.*
Zero Marks :	*0*	*If none of the options is chosen (i.e. the question is unanswered).*
Negative Marks :	*–1*	*In all other cases.*

- ***For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option (i.e. the question is unanswered) will get 0 marks ; and choosing any other combination of options will get –1 mark.***

5. A 470 kg communication satellite is released from a space shuttle at a height of 280 km above the surface of the earth. From this height a rocket engine boosts it into a geosynchronous orbit. The correct statement(s) is/are
(Given that mass of the earth = 5.98×10^{24} kg and radius of the earth = 6400 km)

(a) The orbital radius of geosynchronous orbit of satellite is 4.23×10^7 m
(b) Energy supplied by engine to change the orbit is 1.18×10^{10} J
(c) Change in potential energy of the satellite is -2.36×10^{10} J
(d) Firing of engine results in decrease in potential energy and increase in kinetic energy of the satellite.

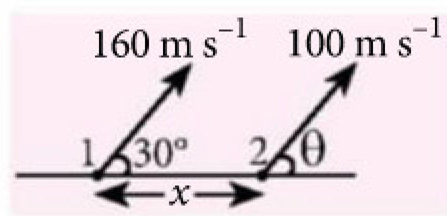
6. A siphon has a uniform circular base of diameter $8/\sqrt{\pi}$ cm with its crest A, 1.8 m above the water level. Vessel B is of large cross section as shown in figure ($g = 10$ m s^{-2} and atmospheric pressure $P_0 = 10^5$ N m^{-2}).



If water starts flowing from vessel to ground through siphon, then

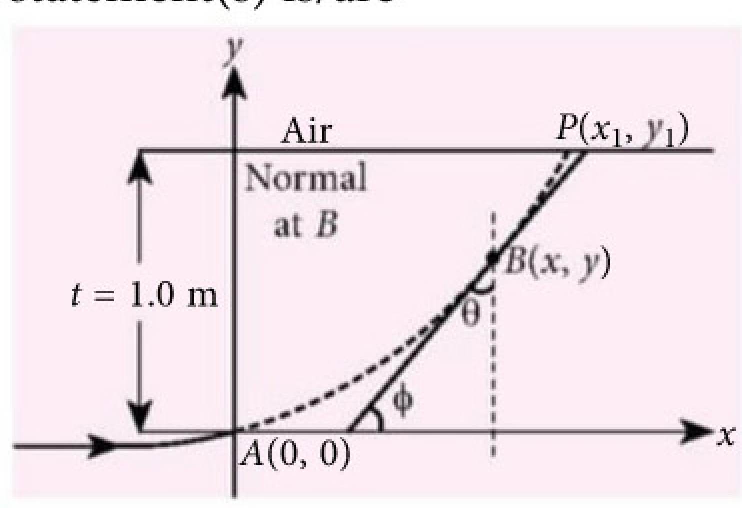
(a) velocity of flow of water through pipe is $6\sqrt{2}$ m s^{-1}.
(b) discharge rate of flow through pipe is $96\sqrt{2} \times 10^{-4}$ m^3 s^{-1}.
(c) velocity of flow of water through pipe is 6 m s^{-1}.
(d) pressure of A is 0.46×10^5 N m^{-2}.

7. Suppose two particles 1 and 2 are projected in vertical plane simultaneously.
Their angles of projection are 30° and θ respectively with the horizontal as shown. Suppose they collide after a time t in air. Then,



(a) $\theta = \sin^{-1}(4/5)$ and they will have same speed just before the collision.
(b) $\theta = \sin^{-1}(4/5)$ and they will have different speed just before the collision.
(c) $x < (1280\sqrt{3} - 960)$ m.
(d) it is possible that the particles collide when both of them are at their highest point.

8. A ray of light travelling in air is incident at grazing angle (angle of incidence ≃ 90°) on a long rectangular slab of a transparent medium of thickness t = 1.0 m. The point of incidence is the origin A(0, 0). The medium has a variable index of refraction given by $n(y) = (ky^{3/2} + 1)^{1/2}$, where $k = 1.0$ m$^{-3/2}$. The refractive index of air is 1. The correct statement(s) is/are



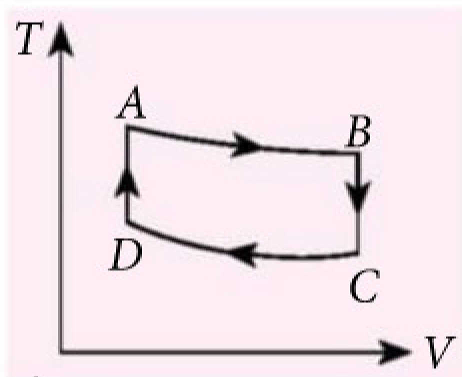
(a) Relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incidence angle at that point is $\cot\theta = \dfrac{dy}{dx}$.

(b) Equation for the trajectory $y(x)$ of the ray in the medium is $4y^{1/2} = \sqrt{k}\,x + C$.

(c) The coordinates (x_1, y_1) of the point P where the ray intersects the upper surface of the slab-air boundary is (4, 1).

(d) The path of the ray will be perpendicular to the boundary.

9. One mole of a monatomic ideal gas is taken through the cycle shown in figure.



$A \to B$ adiabatic expansion

$B \to C$ cooling at constant volume

$C \to D$ adiabatic compression

$D \to A$ heating at constant volume

The pressures and temperatures at A, B, etc., are denoted by P_A, T_A, P_B, T_B, etc., respectively. Given $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$.

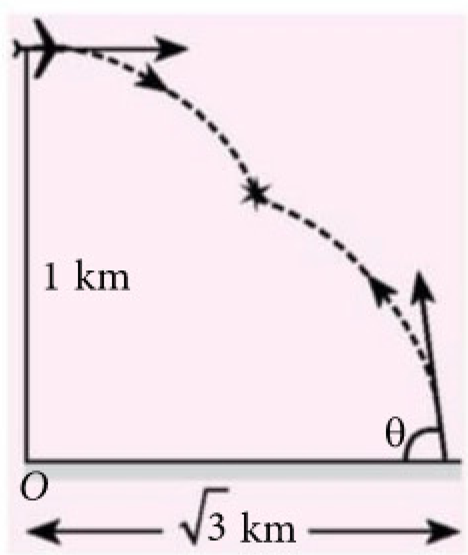
(a) The work done by the gas in the process $A \to B$ is 1869.75 J.

(b) The heat lost by the gas in the process $B \to C$ is – 5297.625 J.

(c) Temperature T_D is 500 K.

(d) Work done from $B \to C$ is 40 J.

10. An aircraft is flying horizontally with a constant velocity 200 m s^{-1}, at a height 1 km above the ground. At the moment shown, a bomb is released from the aircraft and the cannon-gun below fires a shell with initial speed 200 m s^{-1}, at some angle θ.



Both the bomb and the shell collide with each other in air, then

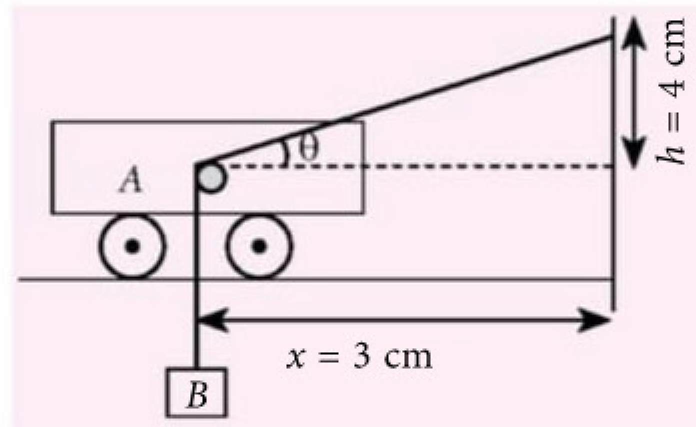
(a) the value of θ at which the projectile shell destroy the bomb in mid-air is 60°.

(b) position of the collision w.r.t. O is $\left(\frac{2}{3}\sqrt{3}, 1 - \frac{\sqrt{3}}{6}\right)$.

(c) they will collide after 10 s.

(d) the value of θ at which the projectile shell destroy the bomb in mid-air is 45°.

11. The string shown in figure is passing over small smooth pulley rigidly attached to trolley A. If the speed of trolley is constant and equal to v_A towards right, speed and magnitude of acceleration of block B at the instant shown in figure are v_B and a_B, then



(a) $v_B = v_A$, $a_B = 0$ (b) $a_B = 0$

(c) $v_B = \frac{3}{5}v_A$ (d) $a_B = \frac{16v_A^2}{125}$

12. A thermostated chamber at a height h above earth's surface maintained at 30°C has a clock fitted with uncompensated pendulum. The maker of the clock for chamber mistakenly designed it to maintain correct time at 20°C. It is found that when the chamber is brought to earth's surface the clock in it clicked correct time. R_e is the radius of Earth.

(a) The variation of length with temperature (of the pendulum) is $\frac{2l}{g}\Delta g$

(b) The coefficient of linear expansion of the material of the pendulum is $\frac{h}{5R_e}$

(c) The coefficient of linear expansion of the material of the pendulum is $\frac{5R_e}{h}$

(d) The variation of length with temperature of the pendulum is $\frac{l}{2g}\Delta g$.

SECTION 3 (Maximum Marks : 18)

- ***This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks : ***+3*** ***If ONLY the correct numerical value is entered.***

Zero Marks : ***0*** ***In all other cases.***

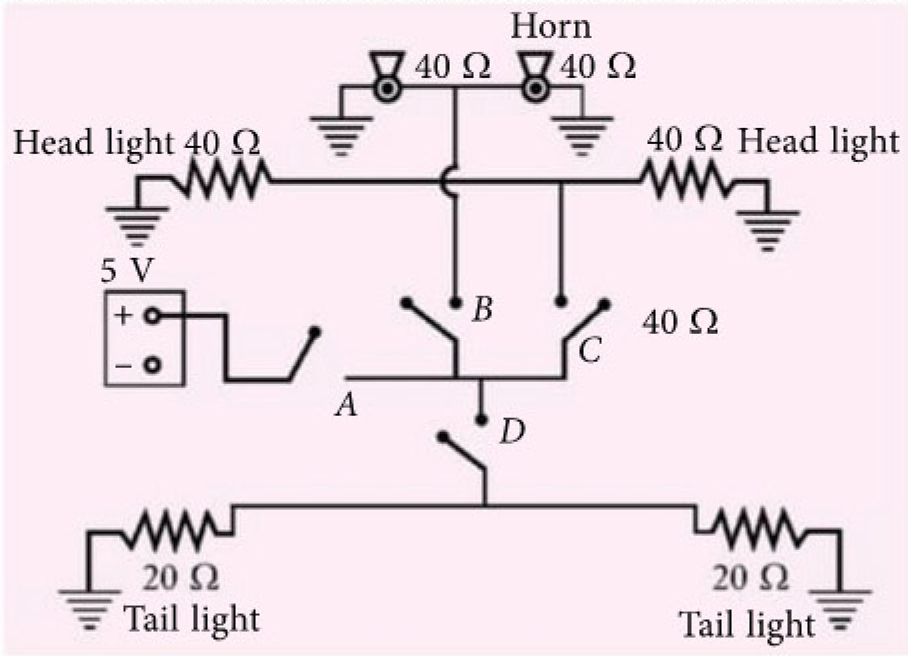
13. In a certain hypothetical radioactive decay process, species A decays into species B and species B decays into species C according to the reactions :

$A \to 2B$ + particles + energy

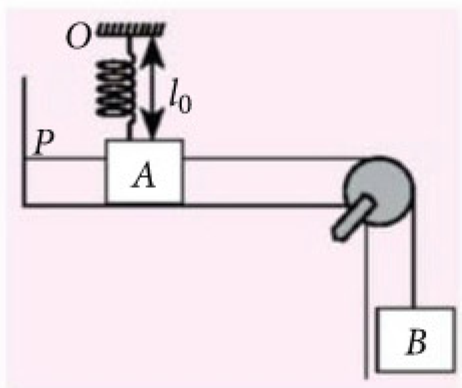
$B \to 3C$ + particles + energy

The decay constant for species A is λ_1 and that for species B is λ_2. Initial there was 10^x moles species A while there was none of B and C. It was found that species B reaches its maximum number at a time t_0. Find the value of x. (Take $\lambda_1 = 1$ dps, $\lambda_2 = 100$ dps, $N_B = 2$ moles at $t_0 = 2 \ln 10$ s)

14. Figure shows an automobile circuit. How much power (in watt) is dissipated by the automobile circuit when switches *A*, *B*, *C* and *D* are all closed?



15. A small bar *A* resting on a smooth horizontal plane is attached by threads to a point *P* and by means of weightless pulley, to a weight *B* possessing the same mass as the bar itself.

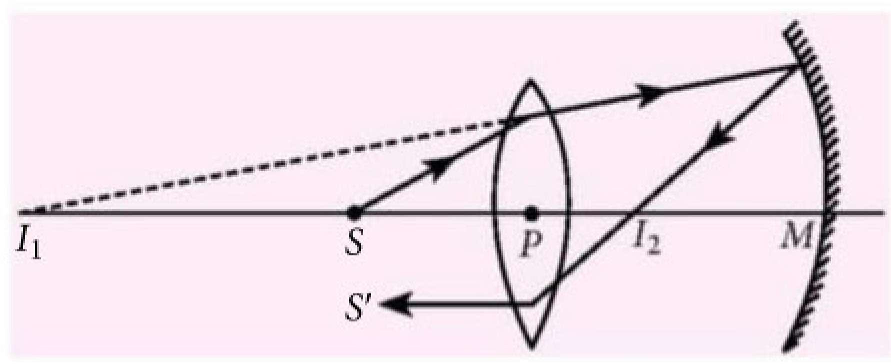


The bar is also attached to a point *O* by means of a light non-deformed spring of length l_0 and stiffness $k = 5\,mg/l_0$, where *m* is the mass of the bar. Now the thread *PA* is burnt and the bar starts moving to the right. Its velocity at the moment when bar is breaking off the plane is given as $\sqrt{\frac{(17+n)gl_0}{16n}}$. Find the value of *n*.

16. End *A* of a rod *AB* of length $L = 0.5$ m and of uniform cross-sectional area is maintained at same constant temperature. The heat conductivity of the rod is $k = 17$ J $s^{-1}m^{-1}K^{-1}$. The other end *B* of this rod is radiating energy into vacuum and the wavelength with maximum energy density emitted from this end is $\lambda_0 = 75000$ Å. If the emissivity of the end *B* is $e = 1$, the temperature of the end *A* is given as $2.11a \times 10^a$ K. (Assuming that except the ends, the rod is thermally insulated) Find the value of *a*.

17. When the soap bubble of radius $R = 0.25$ cm is charged, it experiences an outward electric pressure of magnitude $\frac{\sigma^2}{2\varepsilon_0}$ where its surface charge density $\sigma = 20\ \mu C\ m^{-2}$, If *Q* is the charge on the sphere so that the pressure inside and outside is same, then the surface tension of soap in terms of $\alpha \times 10^{-12}/8\varepsilon_0$ N m^{-1}. Find the value of α.

18. A converging lens of focal length 15 cm and a converging mirror of focal length 20 cm are placed with their principal axes coinciding. A point source *S* is placed on the principal axis at a distance of 12 cm from the lens as shown in figure. It is found that the final beam comes out parallel to the principle axis. Find the separation between the mirror and the lens.



PAPER-II

SECTION 1 (Maximum Marks : 32)

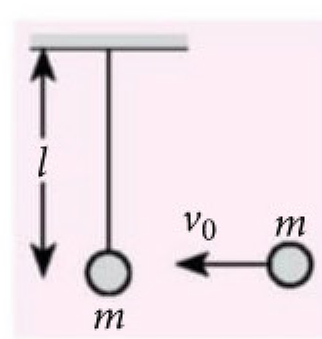
- ***This section contains EIGHT questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct.***
- ***For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.***
- ***For each question, marks will be awarded in one of the following categories :***

Full Marks :	*+4*	*If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.*
Partial Marks :	*+1*	*For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*
Zero Marks :	*0*	*If none of the bubbles is darkened.*
Negative Marks :	*–2*	*In all other cases.*

- ***For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks; and darkening (a) and (b) will result in –2 marks, as a wrong option is also darkened.***

1. *n* drops of a liquid each with surface energy *E* join to form a single drop. Then
(a) Some energy will be released in the process.
(b) Some energy will be absorbed in the process.
(c) The energy released will be $E(n - n^{2/3})$.
(d) The energy absorbed will be $nE(2^{2/3} - 1)$.

2. A simple pendulum consists of a bob of mass *m* and a light string of length *l* as shown in the figure. Another identical ball moving with the small velocity



v_0 collides with the pendulum's bob and sticks to it. If the new pendulum has mass $2m$. Then

(a) Time period of the new pendulum is $2\pi\sqrt{\frac{l}{g}}$.

(b) The equation of motion for the new pendulum is $\theta_0 = \frac{v_0}{2\sqrt{gl}}$.

(c) The equation of motion for the new pendulum is $\theta = \frac{v_0}{2\sqrt{gl}}\cos\left[\sqrt{\frac{g}{l}}\,t\right]$.

(d) Time period of the new pendulum is $2\pi\sqrt{\frac{2l}{g}}$.

3. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(a) $\beta_2 > \beta_1$

(b) $m_1 > m_2$

(c) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(d) The angular separation of fringes for λ_1 is greater than λ_2.

4. A sound wave of frequency υ travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c. Then

(a) The number of wave striking the surface per second is $\upsilon\frac{(c+v)}{c}$

(b) The wavelength of reflected wave is $\frac{c(c-v)}{\upsilon(c+v)}$

(c) The frequency of the reflected wave is $\upsilon\frac{(c+v)}{(c-v)}$

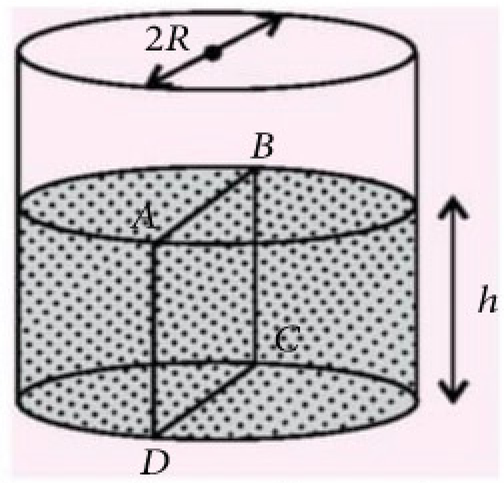
(d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{v\upsilon}{c-v}$

5. A series R-C circuit is connected to an ac voltage source. Consider two cases : (A) when C is without a dielectric medium and (B) when C is filled with a dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$

(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

6. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude



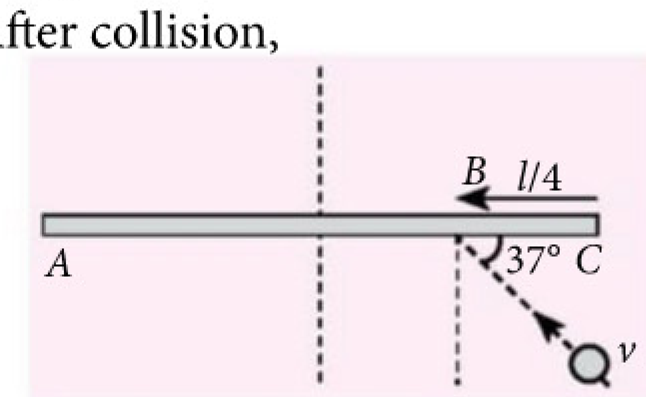
(a) $|2P_0Rh + \pi R^2\rho gh - 2RT|$

(b) $|2P_0Rh + R\rho gh^2 - 2RT|$

(c) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$

(d) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$.

7. A rod AC of length l and mass m is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass m moving on the plane with velocity v strikes the rod at point B making angle 37° with the rod. The collision is elastic. After collision,



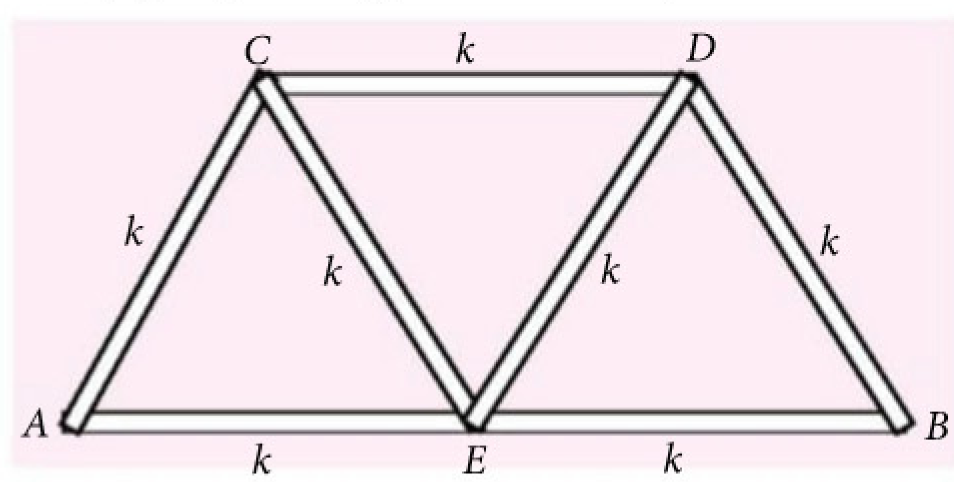
(a) The angular velocity of the rod will be $72/55\ v/l$

(b) The centre of the rod will travel a distance $\pi l/3$ in the time in which it makes half rotation

(c) Impulse of the impact force is $24mv/55$

(d) None of these.

8. Seven identical rods of material of thermal conductivity k are connected as shown in the figure. All the rods are of identical length l and cross-sectional area A. If the one end A is kept at 100°C and the other end B is kept at 0°C, what would be the temperatures of the junctions C, D and E (θ_C, θ_D and θ_E) in the steady state?



(a) $\theta_C > \theta_E > \theta_D$

(b) $\theta_C = 25°C$, $\theta_D = 37.5°C$ and $\theta_E = 50°C$

(c) $\theta_C = 62.5°C$, $\theta_D = 37.5°C$ and $\theta_E = 50°C$

(d) $\theta_C = 60°C$, $\theta_D = 40°C$ and $\theta_E = 50°C$

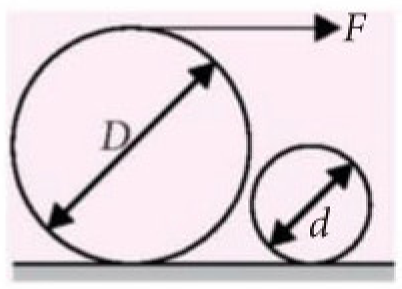
SECTION 2 (Maximum Marks : 18)

- ***This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks : ***+3*** ***If ONLY the correct numerical value is entered.***

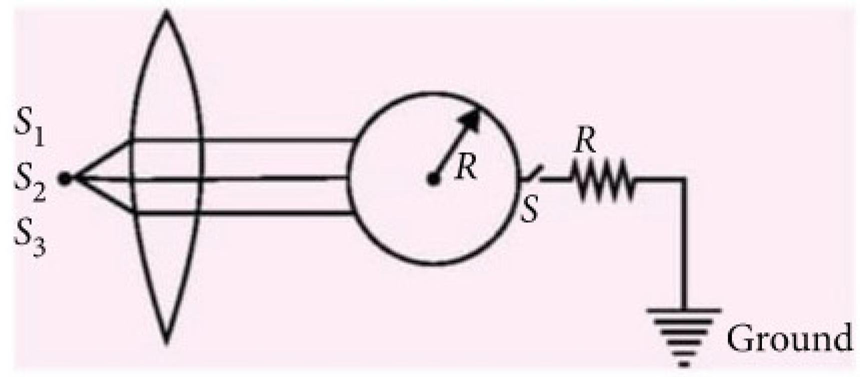
Zero Marks : ***0*** ***In all other cases.***

9. Two cylindrical rollers of diameters D and d respectively rest on a horizontal plane as shown in figure. The diameter of the larger roller is four times that of smaller one. The larger roller wound round with a string is pulled with a horizontal force F. Assuming that the coefficient of friction is μ for all surfaces of contact, find the larger value of μ (in 10^{-1}) as the larger roller can be pulled over the smaller one.

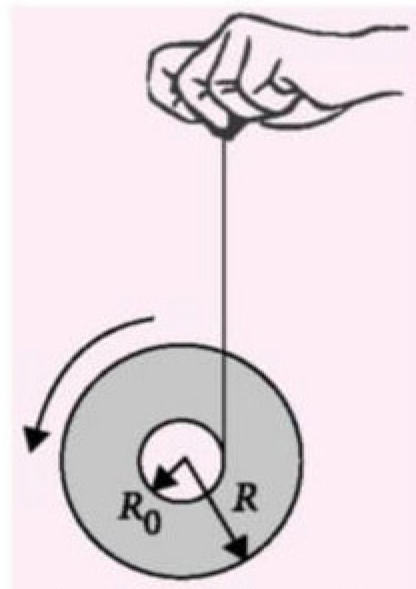


10. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of the thread $L >> a$). One of the balls is then discharged. Again for the certain value of distance b ($b << L$) between the balls, the equilibrium is restored. Find the value of (a^3/b^3).

11. Three monochromatic sources having wavelengths 12.42 nm, 6.21 nm and 24.84 nm are placed close to each other in front of a converging lens such that equal powers from the three sources (equal to 1mW each) fall on a converging lens and then on a small spherical conductor of radius r = 1 mm and its work function = 62.1 eV. Assume 50% efficiency of emission and no change in effective value of work function due to photoelectric emission. Also, all electrons emitted by the sphere are immediately removed by some non-electrical mechanism and its potential rises. At t = 8 s, switch S is closed so that the sphere gets connected to the earth via a resistor of value 6.75 MΩ. If the current flowing just after the switch is closed is given by $10a^3$ A, then find the value of a. (Use, hc = 1242 eV nm)



12. A toy yo-yo of total mass M = 0.24 kg consists of two disks of radius R = 2.8 cm connected by a thin shaft of radius R_0 = 0.25 cm. A string of length L = 1.2 m is wrapped around the shaft. If the yo-yo is thrown downward with an initial velocity of v_0 = 1.4 m s^{-1}, what is its rotational velocity (in rad s^{-1}) when it reaches the end of the string?



A yo-yo falls as the string unwinds from the axle.

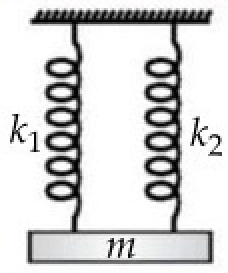
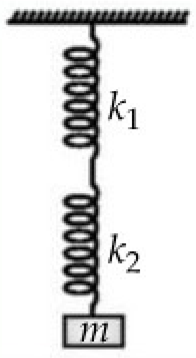
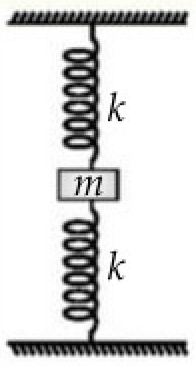
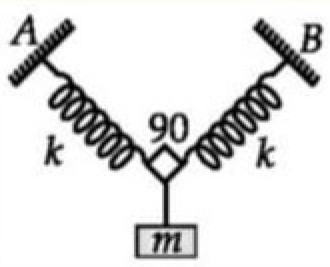
13. A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and 47th circular division coincides with the main scale. Find the curved surface area of wire (in cm^2) to appropriate significant figure.

14. Two identical wires are stretched by the same tension of 100 N, and each emits a note of frequency 200 cycles s^{-1}. The tension in one wire is increased by 1 N. Find the number of beats heard in cycle per second when the wires are plucked.

Section 3 (Maximum Marks : 12)

- ***This section contains FOUR (04) match the columns type questions.***
- ***Each question has FOUR options. ONLY ONE of these four options corresponds to the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***
- ***Full Marks :*** ***+3*** ***If ONLY the correct option is chosen.***
- ***Zero Marks :*** ***0*** ***If none of the options is chosen (i.e. the question is unanswered).***
- ***Negative Marks : –1*** ***In all other cases.***

15. Match List I with List II and select the correct answer using codes given below the Lists.

	List I		List II
P.		1.	$T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$
Q.		2.	$T = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$
R.		3.	$T = 2\pi\sqrt{\frac{m}{k}}$
S		4.	$T = 2\pi\sqrt{\frac{m}{2k}}$

Codes

	P	Q	R	S
(a)	1	4	3	2
(b)	2	3	4	1
(c)	2	1	3	4
(d)	4	1	2	3

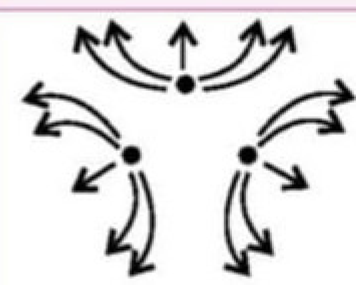
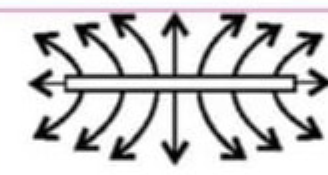
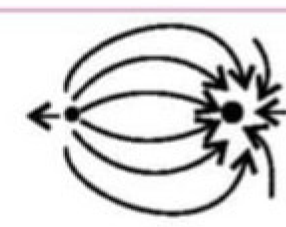
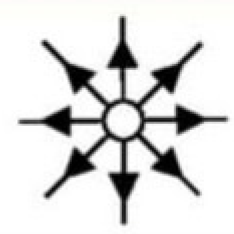
16. Match the quantities given in List I with the horizontal range given in List II for a particle projected with velocity u.

	List I		List II
P.	Its horizontal range is thrice the greatest height attained	1.	$\frac{4}{5}\frac{u^2}{g}$
Q.	Its maximum height is thrice the horizontal range attained	2.	$\frac{24}{145}\frac{u^2}{g}$
R.	Its horizontal range is twice the greatest height attained	3.	$\frac{16}{65}\frac{u^2}{g}$
S.	Its maximum height is twice the horizontal range attained	4.	$\frac{24}{25}\frac{u^2}{g}$

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	2	4	1	3
(c)	4	2	1	3
(d)	4	2	3	1

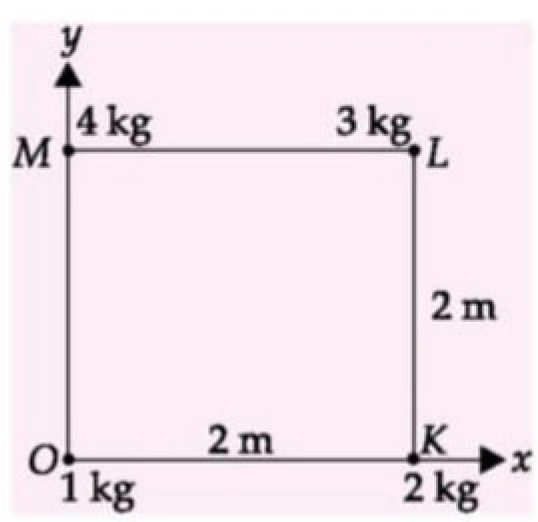
17. Match the List I with List II and select the correct answer using codes given below the lists.

	List I		List II
P.	Three equal positive charges placed at the vertices of an equilateral triangle	1.	
Q.	In a thin, circular disc of uniform positive charge distribution	2.	
R.	Two point charges of $+q$ and $-2q$ separated by a small distance	3.	
S.	A uniformly charged hollow circular cylinder	4.	

Codes

	P	Q	R	S
(a)	4	3	2	1
(b)	3	4	1	2
(c)	1	2	4	3
(d)	2	1	3	4

18. Four point masses of 1 kg, 2 kg, 3 kg and 4 kg are placed at four corners of a square of each side 2 m as shown in figure.



Match the List I with List II and select the correct answer using codes given below the lists.

List I		List II	
P.	x-coordinate of centre of mass of 1 kg and 3 kg	1.	$\frac{2}{3}$ m
Q.	x-coordinate of centre of mass of 2 kg and 4 kg	2.	$\frac{14}{9}$ m
R.	y-coordinate of centre of mass of 1 kg, 2 kg and 3 kg	3.	1 m
S.	y-coordinate of centre of mass of 2 kg, 3 kg and 4 kg	4.	$\frac{3}{2}$ m

Codes

	P	Q	R	S
(a)	4	1	3	2
(b)	1	2	3	4
(c)	1	3	2	4
(d)	4	3	2	1

SOLUTIONS

PAPER-I

1. (a) : From the given condition,

$$OS_3 = \frac{D\lambda}{d} = \frac{1 \times 6 \times 10^{-7}}{3 \times 10^{-3}} = 2 \times 10^{-4} \text{ m}$$

Let light reaching from S_1 and S_2 to S_4 has phase difference ϕ and intensity of incident light is I_0.

Resultant intensity at S_4, $I = 4I_0 \cos^2 \frac{\phi}{2}$

As $I = I_0$, hence, $\frac{I_0}{4I_0} = \cos^2 \frac{\phi}{2}$ or $\cos \frac{\phi}{2} = \frac{1}{2} = \cos 60°$

or $\phi = \frac{2\pi}{3}$

For $\phi = \frac{2\pi}{3}$, $OS_4 = \frac{D\lambda}{3d}$

Therefore, $S_3S_4 = OS_3 + OS_4 = \frac{4}{3}\frac{D\lambda}{d} = \frac{8}{3} \times 10^{-4}$ m

Now resultant wave coming out of S_3 has intensity $4I_0$ and waves coming out of S_4 have intensity I_0.

Phase difference at S_3 = 2π, phase difference at $S_4 = 2\pi/3$.

These phase differences are relative to the light incident on slits S_1 and S_2.

Now S_3 and S_4 are secondary sources of light.

Phase difference at O' = $\frac{4\pi}{3}$, equal to initial phase difference between the light reaching at O'

$$= 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}.$$

Let intensity at O' be I'.

$$I' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0} \cos \frac{4\pi}{3} = 3I_0$$

For brightest fringe, $\phi = 2n\pi$, $n = 0, \pm1, \pm2,....$

Let I'' be the intensity of brightest fringe.

$I'' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0} \cos\phi = 9I_0$ (where $\cos\phi = 1$)

2. (d) **3. (a)**

4. (d) : $E2\pi l = \pi R^2\left(\frac{dB}{dt}\right)$; $E = \frac{R^2}{2l}\left(\frac{dB}{dt}\right)$

$qE + mg = kx$

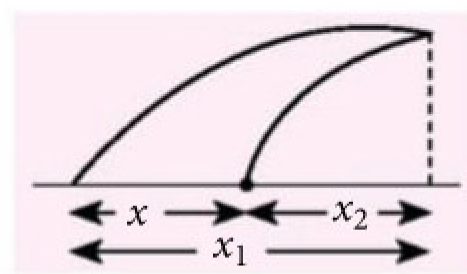
$$\Rightarrow x = \frac{qR^2}{k2l}\left(\frac{dB}{dt}\right) + \frac{mg}{k}; x = \frac{1}{k}\left[mg + \frac{qR^2}{2l}\frac{dB}{dt}\right]$$

5. (a, b) **6. (a, b, d)**

7. (b, c, d) : If they collide, their vertical components of velocities should be same *i.e.*,

$100 \sin\theta = 160 \sin 30°$ $\Rightarrow$ $\sin\theta = 4/5$

Their vertical components will always be same. Horizontal components :



$160 \cos 30° = 80\sqrt{3}$ m s^{-1}

and $100 \cos\theta = 100 \times 3/5$

$= 60$ m s^{-1}

They are not same, hence their velocities will not be same at any time. So (b) is correct.

$$x = x_1 - x_2 = 160 \cos 30° t - 100 \cos\theta\, t$$

$$\Rightarrow x = (80\sqrt{3} - 60)t$$

Time of flight, $T = \frac{2 \times 160 \times \sin 30°}{g} = 16$ s

Now $t < T$ as they to collide in air

$$\Rightarrow \frac{x}{80\sqrt{3} - 60} < 16 \Rightarrow x < (1280\sqrt{3} - 960) \text{ m}$$

Since their times of flight are same, they will simultaneously reach their maximum height. So, it is possible to collide at highest point for certain values of x.

8. (a, c) : The slope of tangent at point B is, $\tan\phi = \frac{dy}{dx}$

The angle of incidence at B is $\theta = 90° - \phi$.

Hence, $\tan(90° - \theta) = \frac{dy}{dx}$ $\Rightarrow$ $\cot\theta = \frac{dy}{dx}$ ...(i)

From Snell's law at A and B, we have

$$1\sin 90° = n(y)\sin\theta,\ \sin\theta = \frac{1}{n(y)} = \frac{1}{(ky^{3/2}+1)^{1/2}}$$

$$\therefore\quad \cot\theta = \frac{\sqrt{1-(1/ky^{3/2}+1)}}{1/(ky^{3/2}+1)^{1/2}} = k^{1/2}y^{3/4} \quad\text{...(ii)}$$

From eqns. (i) and (ii), $\frac{dy}{dx} = \sqrt{k}y^{3/4}$

$$\Rightarrow\quad 4y^{1/4} = \sqrt{k}x + C \quad\text{...(iii)}$$

Now we substitute boundary conditions in eqn. (iii) $x = 0$, $y = 0$, hence $C = 0$. The required trajectory is

$$y = k^2\left(\frac{x}{4}\right)^4$$

At point P, $y = 1$ m, $k = 1.0\ \text{m}^{-3/2}$, we get $x = 4$ m

$\therefore$ The coordinates of P are (4, 1).

From Snell's law, $n_A \sin i_A = n_P \sin i_P$

As $n_A = n_P = 1$ so $i_P = i_A = 90°$

The ray will emerge parallel to boundary.

9. (a, b, c) : For $A \to B$, $T_A^{\gamma}P_A^{1-\gamma} = T_B^{\gamma}P_B^{1-\gamma}$ where $\gamma = 5/3$ for a monatomic gas.

$$T_B = T_A\left(\frac{P_B}{P_A}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850\ \text{K}$$

Work done in an adiabatic process is given by

$$W = \frac{nR(T_A - T_B)}{\gamma - 1} = \frac{1\times 8.31(1000-850)}{(5/3)-1} = 1869.75\ \text{J}$$

$\because$ Process $B \to C$ is isochoric.

Hence $W = 0$ and $\frac{P_C}{T_C} = \frac{P_B}{T_B}$

$$\therefore\quad T_C = \frac{P_C}{P_B}\times T_B = \frac{(1/3)P_A}{(2/3)P_A}T_B = \frac{T_B}{2} = \frac{850}{2} = 425\ \text{K}$$

From first law of thermodynamics,

$$Q = \Delta U + W = nC_V\Delta T + 0 = n\frac{3}{2}R(T_C - 850)$$

Hence, $Q = 1\times\frac{3}{2}\times 8.31(425 - 850) = -5297.625$ J

Negative sign implies that the system has lost heat.

Process $D \to A$ is isochoric

$$\frac{P_D}{P_A} = \frac{T_D}{T_A} \quad\text{or}\quad P_D = P_A\frac{T_D}{T_A}$$

Process $C \to D$ is adiabatic.

Therefore, $\left(\frac{T_D}{T_C}\right)^{\gamma} = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_AT_D}{P_CT_A}\right)^{\gamma-1}$

$$\therefore\quad T_D^{1/\gamma} = T_C\left[\frac{P_A}{P_CT_A}\right]^{1-1/\gamma}$$

$$\Rightarrow\quad T_D^{3/5} = T_C\left[\frac{P_A}{(1/3)P_A\times 1000}\right]^{2/5} = 425\left(\frac{3}{1000}\right)^{2/5}$$

$T_D = 500$ K

10. (a, b) : Suppose the shell destroys the bomb at time t. Then for horizontal motion,

$$t(200 + 200\cos\theta) = \sqrt{3}\times 1000$$

$$\therefore\quad t(1+\cos\theta) = 5\sqrt{3} \quad\text{...(i)}$$

For vertical motion,

$$\frac{1}{2}gt^2 + (200\sin\theta)t - \frac{1}{2}gt^2 = 1000$$

$$\therefore\quad \sin\theta t = 5 \quad\text{...(ii)}$$

From eqns. (i) and (ii), $\frac{\sin\theta}{1+\cos\theta} = \frac{1}{\sqrt{3}}$

On solving, we get $\theta = 60°$.

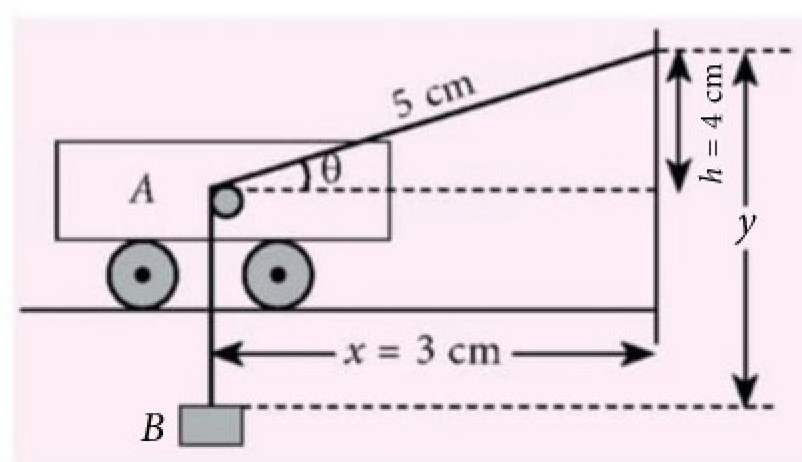
Putting value of θ in eqn. (ii), we get $t = \frac{10\sqrt{3}}{3}$ s

If (x, y) is the coordinate of point of collision,

$$x = 200t = \frac{2000\sqrt{3}}{3}\ \text{m} = \frac{2\sqrt{3}}{3}\ \text{km}$$

$$y = 200\sin\theta t - \frac{1}{2}gt^2 = 1000 - \frac{500\sqrt{3}}{3} = \left(1 - \frac{\sqrt{3}}{6}\right)\text{km}$$

11. (c, d) : $(y - h) + \sqrt{x^2+h^2} = l =$ constant



$$\text{or}\quad \frac{dy}{dt} + \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt} = 0 \ \text{or}\ \frac{dy}{dt} = -\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\therefore\quad \frac{d^2y}{dt^2} = \frac{v_A^2h^2}{(x^2+h^2)^{3/2}} \quad \therefore\ v_B = \frac{3}{5}v_A \text{ and } a_B = \frac{16}{125}v_A^2$$

12. (b) : Variation of acceleration due to gravity with altitude

$$g_h = g\left(1 - \frac{2h}{R_e}\right) \quad\text{or}\quad \Delta g = \frac{2hg}{R_e}$$

$$T_h = 2\pi\sqrt{\frac{l}{g-\Delta g}} = 2\pi\sqrt{\frac{l}{g}}\left(1 - \frac{\Delta g}{g}\right)^{-1/2} = T\left(1 + \frac{\Delta g}{2g}\right)$$

Variation of l with temperature = Δl

$$T_\theta = 2\pi\sqrt{\frac{l+\Delta l}{g}} = T\left(1+\frac{\Delta l}{2l}\right)$$

The clock shows correct time if $T_h = T_\theta$

$$\therefore \quad \frac{\Delta l}{2l} = \frac{\Delta g}{2g}$$

Linear expansivity = $\frac{\Delta l}{l(\Delta\theta)}$, where $\Delta\theta = 30 - 20 = 10°C$

$$\text{Linear expansivity} = \frac{\Delta l}{10l} = \frac{\Delta g}{10g} = \frac{2h}{10R_e} = \frac{h}{5R_e}$$

13. (4) : $\frac{dN_A}{dt} = -\lambda_1 N_A$ and $\frac{dN_B}{dt} = -2\frac{dN_A}{dt} - \lambda_2 N_B$

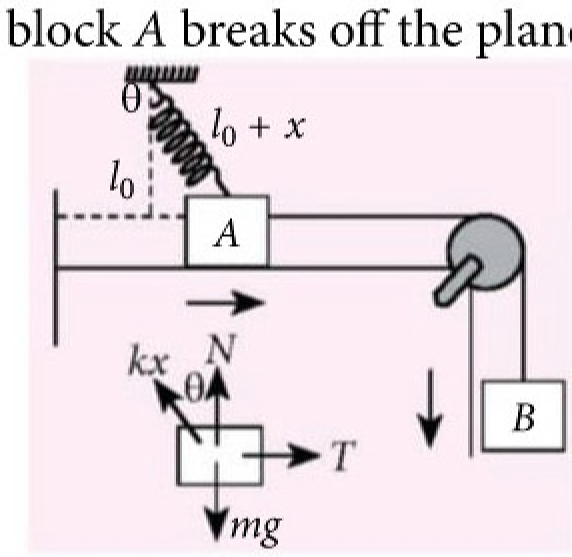
Now when N_B is maximum, at that time

$$\frac{dN_B}{dt} = 0 \quad \Rightarrow \quad 2\lambda_1 N_A - \lambda_2 N_{B\max} = 0$$

or $\quad N_0 = \frac{\lambda_2 N_{B\max}}{2\lambda_1} e^{\lambda_1 t} = 10{,}000$ moles

14. (5)

15. (2) : Let x be the elongation produced in spring and θ be the angle between spring and vertical at the instant when block A breaks off the plane.



$$\Rightarrow \quad \cos\theta = \frac{l_0}{l_0+x} \qquad ...(i)$$

$$N + kx\cos\theta = mg$$

$$\Rightarrow \quad kx\cos\theta = mg \qquad (\text{as } N = 0) \qquad ...(ii)$$

Let d is the distance covered by A and B and v is the speed acquired by them.

From eqns. (i) and (ii), using $k = \frac{5mg}{l_0}$

$$\Rightarrow \quad \frac{5mg}{l_0}\times\frac{l_0 x}{l_0+x} = mg$$

$$\Rightarrow \quad x = \frac{1}{4}l_0 \quad \Rightarrow \quad d = \sqrt{(l_0+x)^2 - l_0^2} = \frac{3l_0}{4}$$

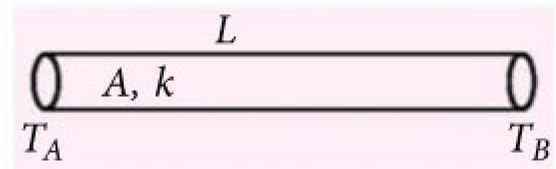
Using energy conservation,

$$mgd = 2\left(\frac{1}{2}mv^2\right) + \frac{1}{2}kx^2$$

$$\Rightarrow \quad mg\frac{3l_0}{4} = mv^2 + \frac{1}{2}\frac{5mg}{l_0}\frac{l_0^2}{16} \quad \Rightarrow \quad v = \sqrt{\frac{19gl_0}{32}}$$

$\therefore$ The value of n is 2

16. (2) : $T_B\lambda_0 = b,\ T_B = \frac{b}{\lambda_0}$



$b = 2.89\times10^{-3}$ m K,

$\lambda_0 = 75000$ Å $= 75\times10^{-7}$ m, $\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}

$$\frac{kA(T_A - T_B)}{L} = \sigma A T_B^4$$

$$T_A = T_B + \frac{L\sigma T_B^4}{k} \Rightarrow T_A = \frac{b}{\lambda_0} + \frac{L\sigma}{k}\times\frac{b^4}{\lambda_0^4}$$

Substituting values, we get $T_A = 422.1$ K $\therefore\ a = 2$

17. (1) : The pressure inside and outside the charged soap bubble is same if $\frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{R};\ T = \frac{\sigma^2 R}{8\varepsilon_0}$

$$T = \frac{\sigma^2 R}{8\varepsilon_0} = \frac{(2\times10^{-5})^2\times0.25\times10^{-2}}{8\varepsilon_0} = \frac{1\times10^{-12}}{8\varepsilon_0}\text{ N m}^{-1}$$

$\therefore$ The value of α is 1.

18. (40) : Let us first locate the image of S formed by the lens L. Here $u = -12$ cm and $f = 15$ cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{15} - \frac{1}{12} \quad \text{or} \quad v = -60 \text{ cm.}$$

The image I_1 acts as the source for the mirror. The mirror forms an image I_2 of the source I_1. This image I_2 then acts as the source for the lens and the final beam comes out parallel to the principal axis. Clearly I_2 must be at the focus of the lens. We have,

$$I_1I_2 = I_1P + PI_2 = 60 \text{ cm} + 15 \text{ cm} = 75 \text{ cm}$$

Suppose the distance of the mirror from I_2 is x cm. For the reflection from the mirror,

$u = MI_1 = -(75 + x)$cm, $v = -x$ cm and $f = -20$ cm.

$$\text{Using} \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{x} + \frac{1}{75+x} = \frac{1}{20}$$

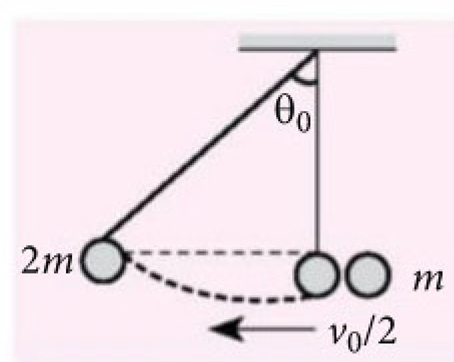
This gives $x = 25$ or -60.

Taking $x = 25$, the separation between the lens and the mirror is $(15 + 25)$ cm = 40 cm

PAPER-II

1. (a, c)

2. (a, b) : The time period of simple harmonic pendulum is independent of mass, so it would be same as that $T = 2\pi\sqrt{l/g}$. After collision, the combined mass acquires



a velocity of $v_0/2$, as a result of this velocity, the mass $2m$ moves up and at an angel θ_0 with vertical, it stops, this is the extreme position of bob.

From work-energy theorem, $\Delta K = W_{total}$

$$0 - \frac{2m}{2}\left(\frac{v_0}{2}\right)^2 = -2mgl(1 - \cos\theta_0)$$

$$\frac{v_0^2}{8gl} = 1 - \cos\theta_0 = 2\sin^2\frac{\theta_0}{2}$$

$$\sin\frac{\theta_0}{2} = \frac{v_0}{4\sqrt{gl}}$$

If θ_0 is small, $\sin\frac{\theta_0}{2} \simeq \frac{\theta_0}{2} \quad \Rightarrow \quad \theta_0 = \frac{v_0}{2\sqrt{gl}}$

3. (a, b, c) : Fringe width, $\beta = \frac{\lambda D}{d}$

$$\beta_1 = \frac{\lambda_1 D}{d} \text{ and } \beta_2 = \frac{\lambda_2 D}{d}$$

$$\therefore \quad \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}; \quad \because \quad \lambda_2 > \lambda_1 \quad \text{so} \quad \beta_2 > \beta_1$$

Number of fringes within a distance y on one side of the central maximum is $m = \frac{y}{\beta}$

Since y is same for λ_1 and λ_2

$$\therefore \quad m_1\beta_1 = m_2\beta_2 \quad \text{or} \quad \frac{m_1}{m_2} = \frac{\beta_2}{\beta_1}$$

$$\because \quad \beta_2 > \beta_1 \quad \text{so} \quad m_1 > m_2$$

3^{rd} maxima of λ_2 is at $= \frac{3\lambda_2 D}{d} = (1800)\frac{D}{d}$

5^{th} minima of λ_1 is at $= (2 \times 5 - 1)\frac{\lambda_1 D}{2d} = (1800)\frac{D}{d}$

So, 3^{rd} maximum of λ_2 overlaps with 5^{th} minimum of λ_1.

Angular separation of fringes, $\theta = \frac{\lambda}{d}$

The angular separation of fringes for λ_1 and λ_2 is

$$\theta_1 = \frac{\lambda_1}{d} \text{ and } \theta_2 = \frac{\lambda_2}{d} \Rightarrow \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2}$$

As $\lambda_1 < \lambda_2 \quad \therefore \quad \theta_1 < \theta_2$

Thus the angular separation of fringes for λ_1 is lesser than λ_2.

4. (a, b, c)

5. (b, c) : Case (A) : $Z_A = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

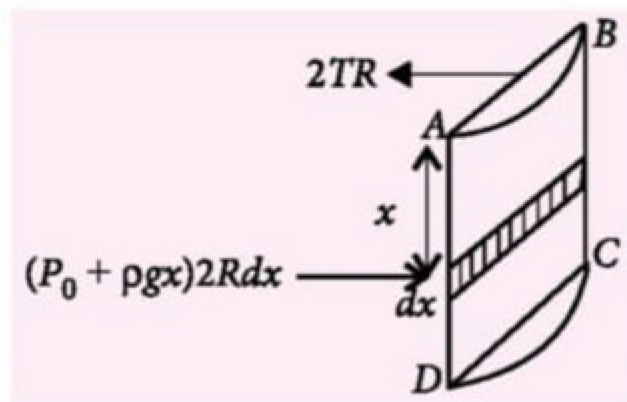
Case (B) : $Z_B = \sqrt{R^2 + \left(\frac{1}{\omega KC}\right)^2}$

So $\quad Z_B < Z_A$

$$I_R^A = \frac{V}{Z_A} \quad \text{and} \quad I_R^B = \frac{V}{Z_B} \quad \text{Clearly } I_R^A < I_R^B$$

Since current in case (B) is greater, so potential across R will increase in case (B) and thus across capacitor will decrease. Hence $V_C^A > V_C^B \qquad (\because V_R^2 + V_C^2 = V_0^2)$

6. (b) :



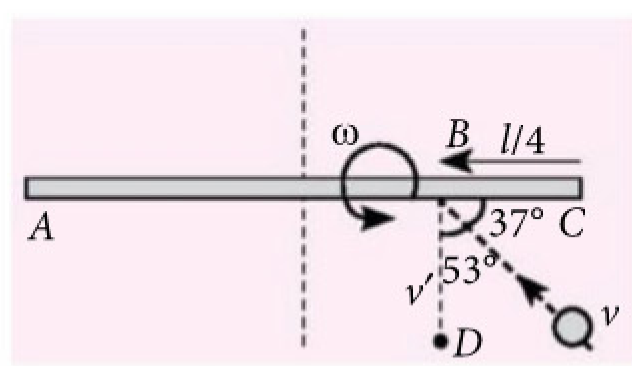
Consider water on one side of the vertical section $ABCD$ as the system. Note that this is the half cylinder. Draw the forces on this half cylinder by another half cylinder. At a depth below x from the top surface consider a strip of width dx. Force on this strip is $(P_0 + \rho gx)(2Rdx)$.

Total force on one half cylinder by the other half cylinder is

$$F = \int_0^h (P_0 + \rho gx)2Rdx - 2RT = 2P_0Rh + R\rho gh^2 - 2RT$$

7. (a, b, c) : The ball has v' component of its velocity perpendicular to the length of the rod immediately after the collision.

u is the velocity of CM of the rod and ω is angular velocity of the rod just after collision. The ball strikes the rod with a speed of $v \cos 53°$ in the perpendicular direction and its component along the length of the rod after the collision is unchanged.



At the point of collision,

velocity of separation = velocity of approach

$$\frac{3v}{5} = \left(\frac{\omega l}{4} + u\right) + v' \qquad \text{...(i)}$$

Conserving linear momentum (of rod + particle) in the direction perpendicular to the rod,

$$mv\frac{3}{5} = mu - mv' \quad ...(ii)$$

Conserving angular momentum about point D,

$$0 = 0 + \left[mu\frac{l}{4} - \frac{ml^2}{12}\omega\right] \Rightarrow u = \frac{\omega l}{3} \quad ...(iii)$$

From eqn. (i), (ii) and (iii)

$$\Rightarrow u = \frac{24v}{55}, \omega = \frac{72v}{55l}$$

Time taken to rotate by π angle, $t = \frac{\pi}{\omega}$

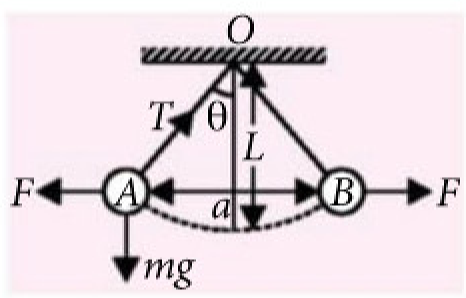
In the same time, distance travelled $= ut = \frac{\pi l}{3}$

Using impulse-momentum equation on the rod,

Impact force $= \int N dt = mu = \frac{24mv}{55}$

8. (a, c) **9. (5)**

10. (4) : Here A and B are two identical balls suspended at a point O as shown in figure.



$$T \sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \quad ...(i)$$

$$T\cos\theta = mg \quad ...(ii)$$

From eqn. (i) and (ii), $\tan\theta = \frac{q^2}{4\pi\varepsilon_0 a^2 mg}$

or $\frac{a}{2L} = \frac{q^2}{4\pi\varepsilon_0 a^2 mg}$ (Since for small θ, $\tan\theta = \frac{a/2}{L}$)

or $\frac{a^3}{L} = \frac{q^2}{2\pi\varepsilon_0 mg}$ ...(iii)

When one of the balls is discharged, the balls come closer and touch each other and again separate due to repulsion. The charge on each ball after touching each other is $q/2$. Replacing q with $q/2$ in (iii), we get

$$\frac{b^3}{L} = \frac{(q/2)^2}{2\pi\varepsilon_0 mg} \quad ...(iv)$$

From eqns. (iii) and (iv), $\frac{a^3}{b^3} = 4$

11. (2) **12. (610)**

13. (2.6) : Least count of screw gauge

$$= \frac{\text{pitch}}{N} = \frac{1\text{ mm}}{100} = 0.01\text{ mm}$$

Diameter $= D = 1 + (47 \times 0.01) = 1.47$ mm $= 0.147$ cm

Area of curved surface, $S = \pi D l$

$$\therefore S = \frac{22}{7} \times 0.147 \times 5.6 = 2.5872\text{ cm}^2$$

Round off to two significant digits $= 2.6\text{ cm}^2$.

14. (1) : The frequency of the fundamental note emitted by each wire before the tension change occurs is

$$\upsilon = \frac{1}{2L}\left(\frac{T}{\mu}\right)^{1/2} \quad ...(i)$$

If T changes, υ will also change. We can find the relation between these two changes by taking the derivative of eqn. (i) w.r.t. T,

$$\frac{d\upsilon}{dT} = \frac{1}{2L}\left[\frac{1}{2}\left(\frac{T}{\mu}\right)^{-1/2}\frac{1}{\mu}\right]$$

$$\frac{d\upsilon}{dT} = \frac{1}{4LT}\left(\frac{T^2\mu}{T\mu^2}\right)^{1/2} = \frac{1}{4LT}\left(\frac{T}{\mu}\right)^{1/2}$$

From eqn. (i), $\frac{d\upsilon}{dT} = \frac{\upsilon}{2T}$

Hence, $\Delta\upsilon \approx \frac{\upsilon}{2}\frac{\Delta T}{T}$

where $\Delta\upsilon$ is the frequency difference induced in the string as a result of a change in tension ΔT. In other words, $\Delta\upsilon$ is the number of beats observed if the string's tension is changed by a small amount ΔT. Using the given data,

$$\Delta\upsilon = \left(\frac{200}{2}\right)\left(\frac{1}{100}\right) = 1\text{ cycle s}^{-1}$$

15. (c) : P-2; Q-1; R-3; S-4

In figure (P), the effective spring constant,

$$K = k_1 + k_2 \Rightarrow T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

In figure (Q), the effective spring constant

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2}$$

or $K = \frac{k_1 k_2}{(k_1 + k_2)} \Rightarrow 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$

In figure (R), the effective spring constant,

$$K = k + k = 2k \Rightarrow T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{2k}}$$

In figure (S), let the effective spring constant be K. If mass m be pulled down by distance y, the springs A and B will be stretched by

$y' = y \cos 45°$

Total restoring force is

$F = -Ky = F_A \cos 45° + F_B \cos 45° = (F_A + F_B)\cos 45°$

$= 2ky' \cos 45°$

$= 2k(y \cos 45°) \cos 45° = ky$

$\therefore \quad K = k$

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \therefore \quad T = 2\pi\sqrt{\frac{m}{k}}$$

16. (c) : (P) As, horizontal range = 3 × greatest height

$$\therefore \quad \frac{u^2 \sin 2\theta}{g} = 3 \times \frac{u^2 \sin^2\theta}{2g}$$

or $\quad 2\sin\theta\cos\theta = \frac{3}{2}\sin^2\theta$

or $\quad \tan\theta = \frac{4}{3}, \sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$

$$\therefore \quad \text{Horizontal range} = \frac{u^2}{g} \times 2\sin\theta\cos\theta$$

$$= \frac{u^2}{g} \times 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24\,u^2}{25\,g}$$

(Q) As, maximum height = 3 × horizontal range

$$\therefore \quad \frac{u^2\sin^2\theta}{2g} = \frac{3u^2 2\sin\theta\cos\theta}{g}$$

or $\quad \tan\theta = 12.$

Now, $\sin\theta = \frac{12}{\sqrt{145}}$ and $\cos\theta = \frac{1}{\sqrt{145}}$

$$\therefore \quad \text{Horizontal range} = \frac{u^2}{g} 2\sin\theta\cos\theta$$

$$= \frac{u^2}{g} \times 2 \times \frac{12}{\sqrt{145}} \times \frac{1}{\sqrt{145}} = \frac{24\,u^2}{145\,g}$$

(R) As horizontal range = 2 × greatest height

$$\Rightarrow \quad \frac{u^2}{g} \times 2\sin\theta\cos\theta = 2 \times \frac{u^2\sin^2\theta}{2g}$$

$\therefore \quad \tan\theta = 2.$

Now $\sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$

$$\therefore \quad \text{Horizontal range} = \frac{u^2}{g} \times 2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4\,u^2}{5\,g}$$

(S) As, maximum height = 2 × horizontal range

$$\therefore \quad \frac{u^2\sin^2\theta}{2g} = 2 \times \frac{u^2}{g} 2\sin\theta\cos\theta$$

$\therefore \quad \tan\theta = 8,$

Now $\sin\theta = \frac{8}{\sqrt{65}}$ and $\cos\theta = \frac{1}{\sqrt{65}}$

$$\therefore \quad \text{Horizontal range} = \frac{u^2}{g} \times 2 \times \frac{8}{\sqrt{65}} \times \frac{1}{\sqrt{65}} = \frac{16\,u^2}{65\,g}$$

17. (c)

18. (a) : P : $m_1 = 1$ kg, $m_2 = 3$ kg, $x_1 = 0$, $x_2 = 2$ m

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1\times 0 + 3\times 2}{1+3} = \frac{3}{2}\text{m}$$

P – 4

Q : $m_1 = 2$ kg, $m_2 = 4$ kg, $x_1 = 2$ m, $x_2 = 0$

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{2\times 2 + 4\times 0}{2+4} = \frac{2}{3}\text{m}$$

Q – 1

R : $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 3$ kg

$y_1 = 0$, $y_2 = 0$, $y_3 = 2$ m

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times 0 + 2\times 0 + 3\times 2}{1+2+3} = 1\text{ m}$$

R – 3

S : $m_1 = 2$ kg, $m_2 = 3$ kg, $m_3 = 4$ kg

$y_1 = 0$, $y_2 = 2$ m, $y_3 = 2$ m

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{2\times 0 + 3\times 2 + 4\times 2}{2+3+4} = \frac{14}{9}\text{ m}$$

S – 2

CLASS-XII

BRUSH UP *for*

NEET/JEE 2021

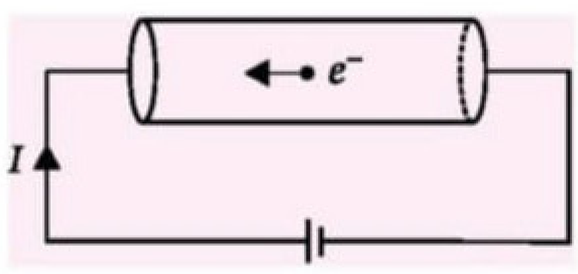
Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 2 Current Electricity

Electric Current and Current Density

- Electric current is the rate of flow of electric charge through a cross section of a conductor

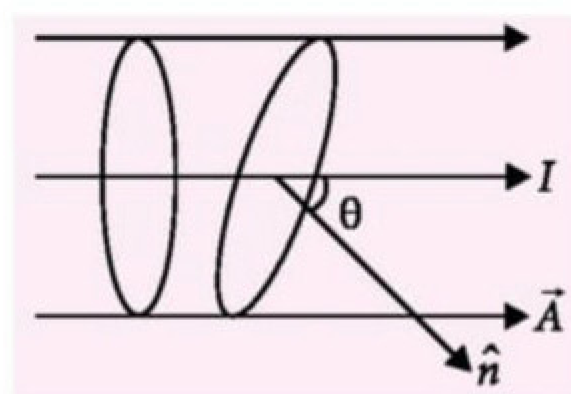
$$I = \frac{dq}{dt}$$



- For steady current, $I = \frac{q}{t}$
- For average current, $I_{av} = \frac{\Delta Q}{\Delta t}$
- Electric current is a scalar quantity because it does not obey the law of vectors. Its SI unit is ampere (A) or coulomb per second (C s^{-1})
- One ampere of current means the flow of 6.25×10^{18} electrons per second through any cross section of conductor.
- Current density at a point in a conductor is the ratio of the current at that point in the conductor to the area of cross section of the conductor of that point,

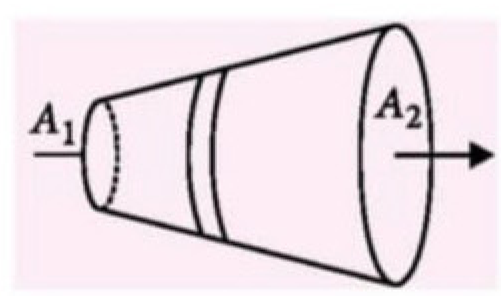
$$\vec{J} = \frac{dI}{dA}\hat{n} \text{ or, } I = \int \vec{J} \cdot d\vec{A}$$

- If A is not normal to I but makes an angle θ with the normal to current, then
$I = JA \cos\theta$



$$J = \frac{I}{A\cos\theta}$$

- Current density is a vector quantity and its SI unit is A m^{-2}.
- If a steady current flows in a metallic conductor of non uniform cross section.

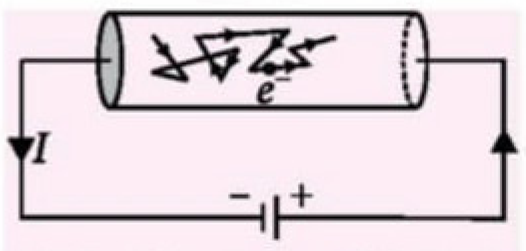


  - Along the wire, current is same, $I_1 = I_2$
  - Current density depends inversely on area, so $J_1 > J_2$, as $A_1 < A_2$

Thermal Velocity and Drift Velocity

- Thermal velocities of free electrons are randomly distributed in all possible direction of a metallic conductor.
- Average thermal velocity is zero but average thermal speed is non zero.
- Drift velocity is defined as the velocity with which

the free electrons get drifted towards the positive terminal under the effect of the applied electric field.



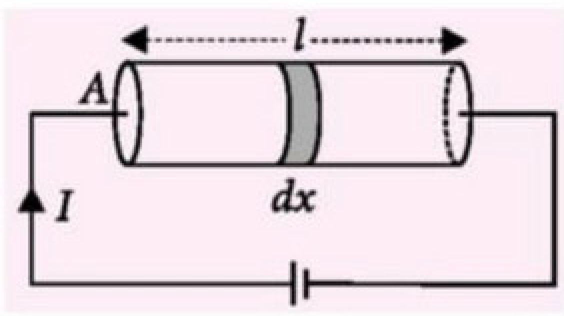
Under the action of electric field : Random motion of an electron with superimposed drift

- $\vec{v}_d = \vec{a}\tau \Rightarrow \vec{v}_d = -\frac{e\vec{E}}{m}\tau$
- Order of drift velocity is 10^{-4} m s^{-1}.
- τ is the relaxation time which is average time elapsed between two successive collisions. It decreases with increase in temperature.

Relation between Current Density, Conductivity and Electric Field

- Let the number of free electrons per unit volume in a conductor = n

  Total number of electrons in dx distance = $n(Adx)$
  Total charge $dQ = n(Adx)e$
  Cross sectional area = A



- Current $I = \frac{dQ}{dt} = nAe\frac{dx}{dt} \Rightarrow I = neAv_d$
- Current density $J = \frac{I}{A} = nev_d \Rightarrow J = ne\left(\frac{eE}{m}\right)\tau$

  $J = \left(\frac{ne^2\tau}{m}\right)E \Rightarrow J = \sigma E$

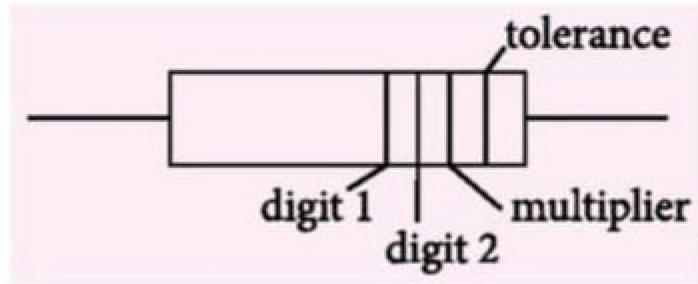
  Conductivity $\sigma = \frac{ne^2\tau}{m}$

  In vector form $\vec{J} = \sigma\vec{E}$ (Another form of Ohm's law)
  σ depends only on the material of the conductor and its temperature.

- Ohm's law, $J = \sigma E \Rightarrow \frac{I}{A} = \frac{1}{\rho}\frac{V}{l} \Rightarrow V = \left(\frac{\rho l}{A}\right)I = RI$
  - At a given temperature, current is directly proportional to the applied potential difference.
  - The substances which obey ohm's law are called ohmic.

Colour Code of Resistors

- A colour code is used to indicate the resistance value and its percentage accuracy.
- The first two coloured rings from the left end indicate the first two significant figures of the resistance in ohms. The third colour ring indicates the decimal multiplier and the last colour ring stands for the tolerance in percent.



- The colour code of a resistor is as shown in the table.

Colour	Number	Multiplier	Tolerance
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
No colour			20%

Resistance and its Combinations

- When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called resistance of the conductor.
- Resistance is the property of a conductor by virtue of which it opposes the flow of current in it.

  $R = \frac{\rho l}{A} = \frac{\rho l^2}{V} = \frac{\rho V}{A^2}$

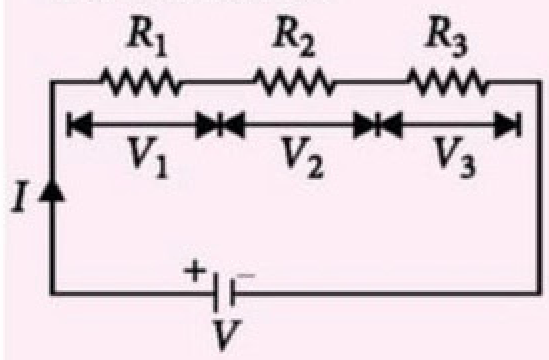
- Resistance of the conductor depends on the temperature and varies as $R_t = R_0\ (1 + \alpha\ \Delta t)$, where R_t = Resistance at t °C, R_0 = resistance at 0 °C, Δt = change in temperature, α = temperature coefficient of resistance.
- For metal : α is positive and for semiconductors and insulators : α is negative.
- Resistance of the conductor decreases linearly with decrease in temperature and becomes zero at

a specific temperature. This temperature is called critical temperature, at this temperature conductor becomes a superconductor.

Combination of Resistance

Series combination

- Current is same across each resistance.



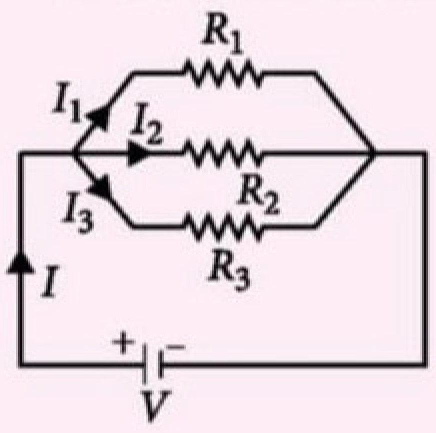
$V = V_1 + V_2 + V_3$

$IR_S = IR_1 + IR_2 + IR_3$

- $R_S = R_1 + R_2 + R_3$
- $R_S > \max(R_1 / R_2 / R_3)$

Parallel combination

- Potential drop across all resistances are same.

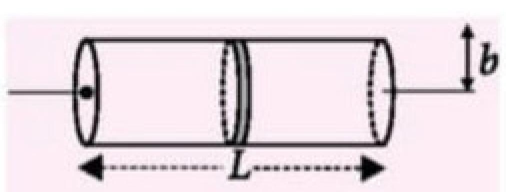


$I = I_1 + I_2 + I_3$

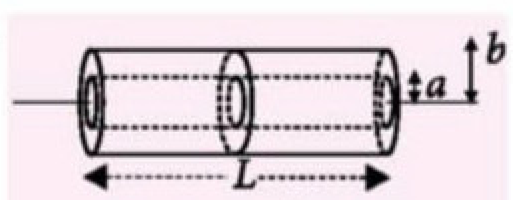
$\frac{V}{R_P} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

- $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
- $R_P < \min(R_1 / R_2 / R_3)$

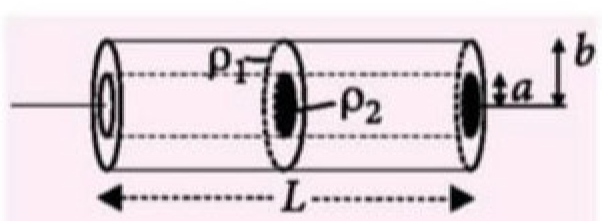
- If a wire is stretched to n times of its original length, its new resistance will be n^2 times.
- If a wire is stretched such that its radius is reduced to $\frac{1}{n^{th}}$ of its original values, then resistance will increase n^4 times similarly resistance will decrease n^4 time if radius is increased n times by contraction.
- Resistance of different shaped conductors.



$R = \frac{\rho L}{\pi b^2}$

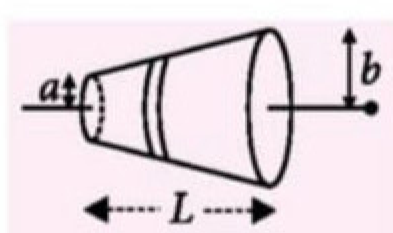


$R = \frac{\rho L}{\pi(b^2 - a^2)}$



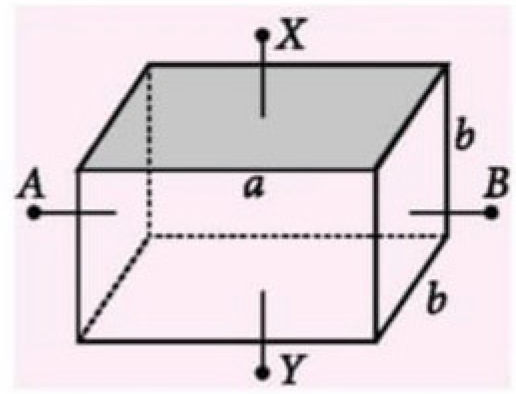
$R_1 = \frac{\rho_1 L}{\pi(b^2 - a^2)}, R_2 = \frac{\rho_2 L}{\pi a^2}$

Net $R = \frac{R_1 R_2}{R_1 + R_2}$



$R = \frac{\rho L}{\pi ab}$

(Here, $(b - a) << L$)



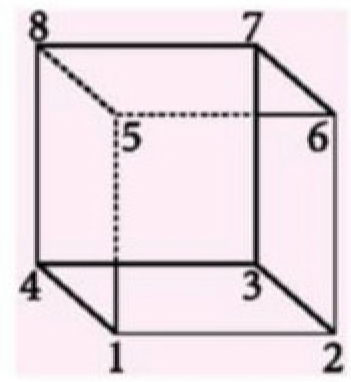
– Resistance between square faces

$$R_{AB} = \rho \frac{\text{distance between faces}}{\text{area of square}} = \rho \frac{a}{b^2}$$

– Resistance between rectangular faces

$$R_{XY} = \rho \frac{b}{a \cdot b} = \frac{\rho}{a} \quad \text{(does not depend on } b\text{)}$$

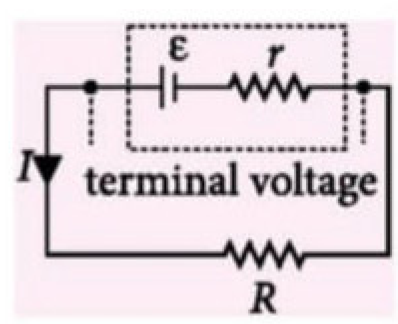
- A frame of cube is made with wires each of equal resistance r then



  - Resistance between two nearer corners $R_{12} = \frac{7}{12} r$
  - Resistance across face diagonal $R_{13} = \frac{3}{4} r$
  - Resistance across main diagonal $R_{17} = \frac{5}{6} r$

EMF (ε) and Terminal Voltage (V)

- The potential difference across the terminals of a cell when it is not producing any current is called emf of the cell.
- The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.
- emf depends on
  - nature of electrolyte
  - metal of electrodes
- emf does not depend on
  - area of plates
  - distance between the electrodes
  - quantity of eletrolyte
  - size of cell
- When current drawn through the cell or current is supplied to cell, the potential difference across its terminals = terminal voltage.



  - When current I is drawn from the cell, then terminal voltage is less than its emf. $V = \varepsilon - Ir$
  - At the time of charging a

cell, current is supplied to the cell, the terminal voltage is greater than emf ε.

$V = \varepsilon + Ir$

- Efficiency of cell is defined by

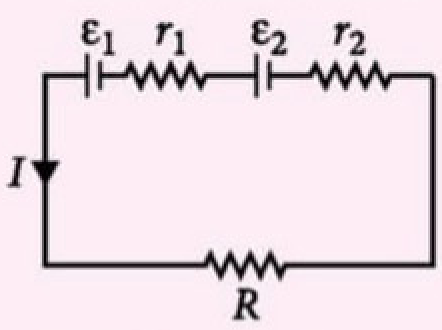
$$\eta = \frac{\text{Total potential difference}}{\text{emf}} \times 100\%$$

- Internal resistance of a cell

  - Resistance offered by the electrolyte of the cell when the electric current flows through it is known as internal resistance.
  - Distance between two electrodes increases $\Rightarrow r$ increases
  - Area dipped in electrolyte increases $\Rightarrow r$ decreases
  - Concentration of electrolyte increases $\Rightarrow r$ decreases
  - Temperature increases $\Rightarrow r$ decreases

Combination of Cells (Battery)

Series Combination



Equivalent emf, $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 = \sum_i \varepsilon_i$

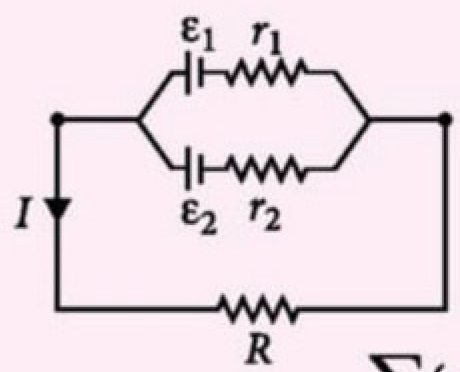
Equivalent resistance,

$r_{eq} = r_1 + r_2 = \sum_i r_i$

Current in the circuit :

$$I = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R}$$

Parallel Combination

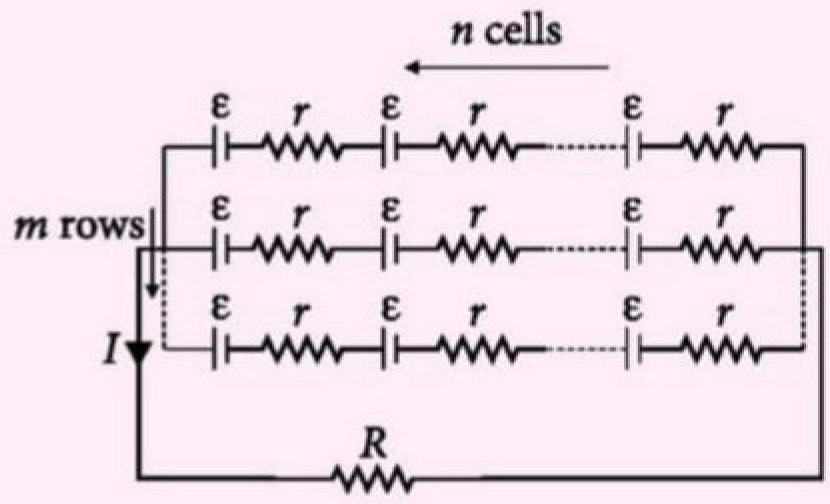


$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} = \frac{\sum_i (\varepsilon_i / r_i)}{\sum_i (1/r_i)}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \Rightarrow \frac{1}{r_{eq}} = \sum_i \left(\frac{1}{r_i}\right)$$

$$I = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

Mixed Combination (Identical cells)



$$\varepsilon_{eq} = n\varepsilon, \quad r_{eq} = \frac{nr}{m}, \quad I = \frac{n\varepsilon}{R + \frac{nr}{m}}$$

☞ Kirchhoff's Laws

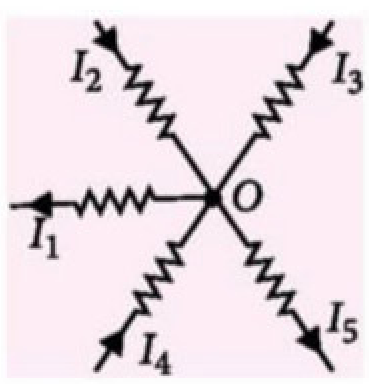
- First law (Kirchhoff's current law)
  - In an electric circuit, the algebraic sum of the current meeting at any junction in the circuit is zero.

$$\sum I = 0$$

  - From the figure,

$I_1 - I_2 - I_3 - I_4 + I_5 = 0$

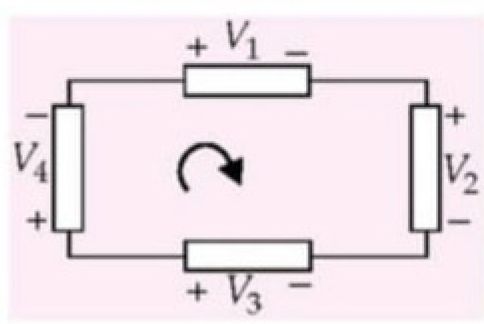
$I_1 + I_5 = I_2 + I_3 + I_4$



  - This is based on law of conservation of charge.
- Second law (Kirchhoff's voltage law)
  - The algebraic sum of all the potential differences along a closed loop is zero.

$\Sigma IR + \Sigma \text{EMF} = 0$

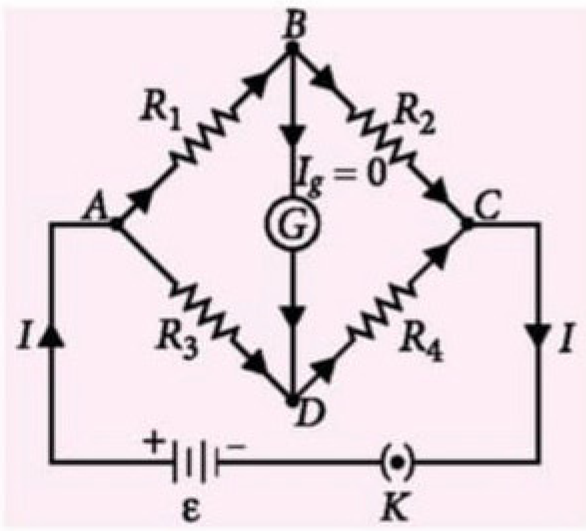
  - The closed loop can be traversed in any direction. While traversing a loop if potential increases, put a positive sign in expression and if potential decreases put a negative sign.



  - From the figure, $-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or nonlinear).
  - This law is based on conservation of energy.

☞ Wheatstone Bridge

- It consists of four resistances R_1, R_2, R_3 and R_4 which are connected to form a quadrilateral $ABCD$ as shown in figure.

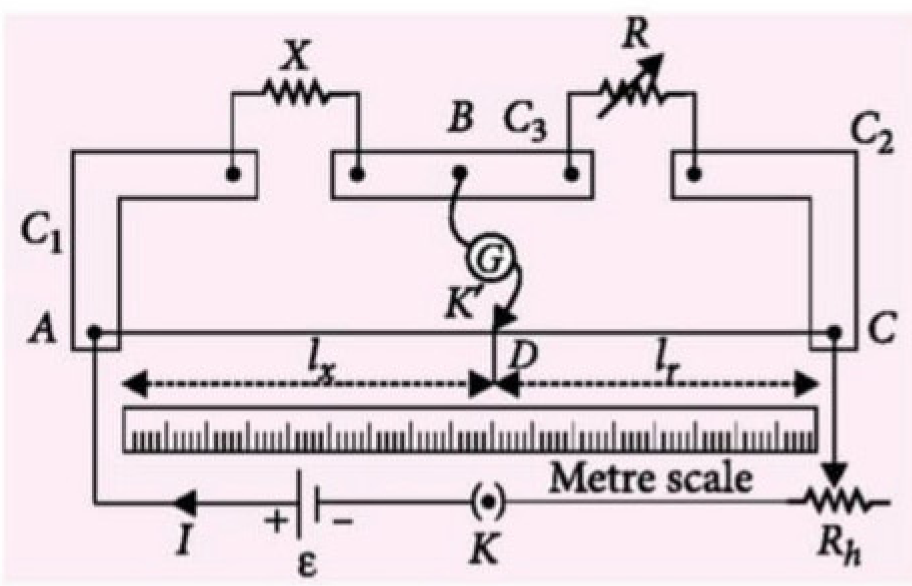


- In a balanced condition even though current flows in the rest of the circuit, galvanometer will not show any deflection (*i.e.* $I_g = 0$).

  Also, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
- The measurement of resistance by Wheatstone's bridge is not affected by the internal resistance of the cell.

☞ Meter-Bridge

- Meterbridge is the modification of Wheatstone's network used to determine unknown resistance. It is an instrument which works on the principle of Wheatstone's network. So it is also called Wheatstone's meterbridge.
- The length of wire used is one metre, so it is called meterbridge.



- From the balancing condition,

$$\frac{X}{R} = \frac{\text{Resistance of wire } AD \text{ of length } l_x}{\text{Resistance of wire } DC \text{ of length } l_r}$$

$$X = R\left(\frac{l_x}{100 - l_x}\right)$$

- Minimization of errors in measurement of X
  - The wire used must be uniform *i.e.*, of same cross section.
  - The value of R should be so chosen that the null point is obtained as near (close) to the centre of wire as possible.
  - The experiment should be repeated by interchanging the positions of X and R to minimize an error due to contact resistance.

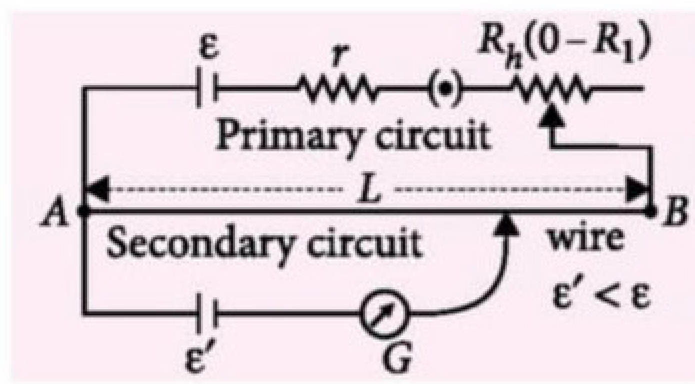
☞ Potentiometer

- Potentiometer is an ideal voltmeter because it draws no current from the circuit at the instant of measurement.
- Working : It is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross-section and a constant current is flowing through it.

  i.e., $V \propto l$ (If I and A are constants.)

  or $V = kl$

  where k is known as potential gradient *i.e.*, fall of potential per unit length of the given wire.
- Circuit of potentiometer



  - Primary circuit contains constant source of voltage, rheostat of resistance box and resistance of the potentiometer of wire.
  - Secondary circuit contains unknown circuit with galvanometer.

  Let ρ = Resistance per unit length of potentiometer wire
- Application of Potentiometer
  - Comparision of emf of two cells

  (i) plug only in (1 –2)

  Jockey is at position J

  balancing length $AJ = l_1$

  $\varepsilon_1 = kl_1$

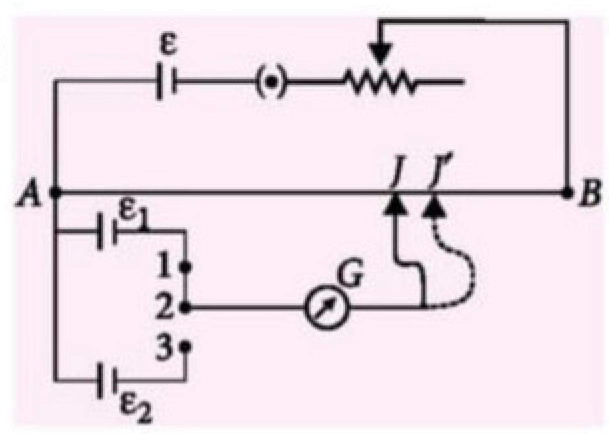
  (ii) Plug only in (2 –3)

  Jockey is at position J'

  balancing length $AJ' = l_2$

  $\varepsilon_2 = kl_2$

  $\therefore \quad \frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$



  - Internal resistance of given primary cell

$$\varepsilon = V + Ir \Rightarrow r = \frac{\varepsilon - V}{I}$$

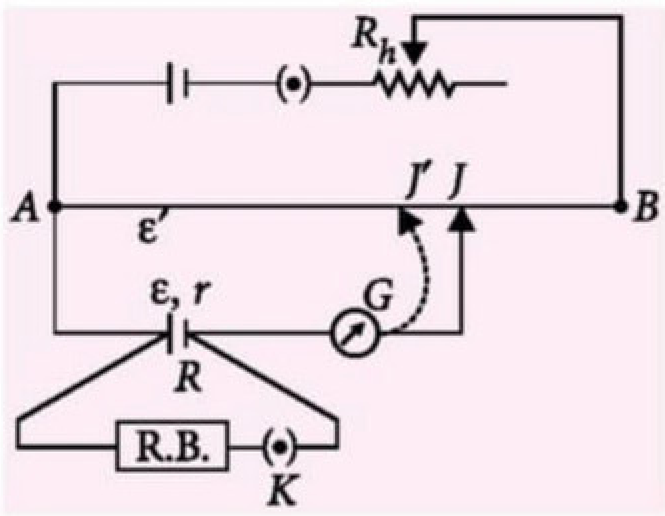
$$\text{or } r = \left(\frac{\varepsilon - V}{V}\right)R$$

Key K open $\varepsilon = kl_1$ $(AJ = l_1)$

Key K closed total potential difference

$V = kl_2$ $(AJ' = l_2)$

$$r = \left(\frac{l_1 - l_2}{l_2}\right)R$$



- The emf of battery (driver cell or auxiliary cell) must be greater than emf's to be compared *i.e.*, $\varepsilon > \varepsilon_1$, $\varepsilon > \varepsilon_2$ and for the combination method $\varepsilon > (\varepsilon_1 + \varepsilon_2)$...
- The positive terminal of ε_1 or ε_2 or of the combination must be connected to that end of potentiometer wire where positive terminal of the battery (driving cell) is connected.
- The potentiometer wire must be uniform.
- The resistance of potentiometer wire should be high.

Joule's Law of Heating

- According to Joule's heating effect of current, the amount of heat produced H in a conductor of resistance R, carrying current I for time t is

$$H = I^2Rt \text{ (in joule)}$$

or $$H = \frac{I^2Rt}{J} \text{ (in calorie)}$$

where J is Joule's mechanical equivalent of heat ($= 4.2$ J cal^{-1}).

Electric Power

- It is defined as the rate at which work is done by the source of emf in maintaining the current in the electric circuit.

Electric power $P = \dfrac{\text{electric work done}}{\text{time taken}}$

$$P = VI = I^2R = \frac{V^2}{R}.$$

- The SI unit of power is watt (W).
- The practical unit of power is kilowatt (kW) and horse power (hp).
- 1 kilowatt = 1000 watt.
- 1 hp = 746 watt.
- If one heater boils a certain mass of water in time t_1 and another heater boils the same mass of water in time t_2, then both the heaters are connected in series, the same water will boil in time $(t_1 + t_2)$; when both the heaters are connected in parallel the same water will boil in time $t = \dfrac{t_1 \times t_2}{t_1 + t_2}$.
- If P_1, P_2, P_3 .... are the powers of electric appliances in series with source of rated voltage V, the effective power consumed is

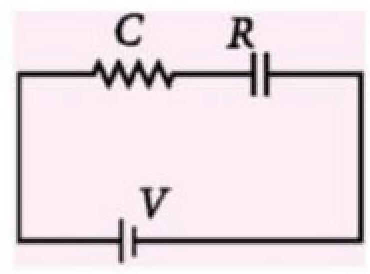
$$\frac{1}{P_S} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + ...$$

- If P_1, P_2, P_3 are the powers of electric appliances in parallel with a source of rated voltage V, the effective power consumed is

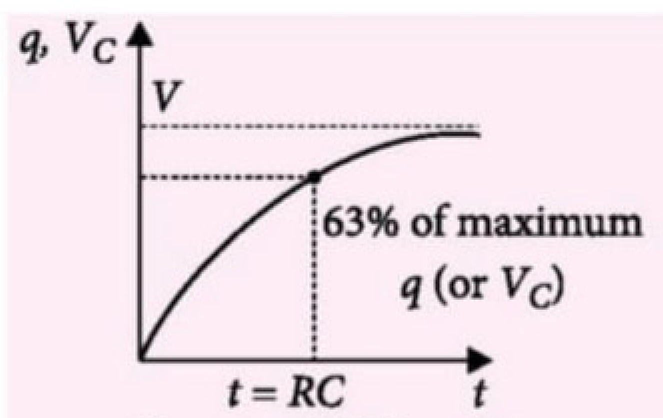
$$P_P = P_1 + P_2 + P_3 + ...$$

Charging and Discharging of Capacitors

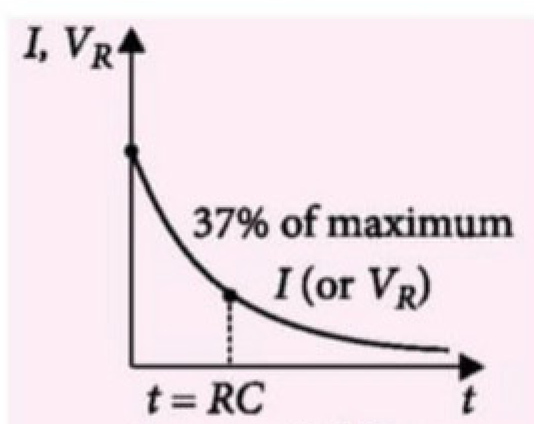
- Charging a capacitor
  - Charge on the capacitor at any time t,
    $q = CV(1 - e^{-t/RC})$
  - Current in the circuit at any time t,
    $I = \dfrac{V}{R}e^{(-t/RC)}$



  - Maximum charge on capacitor, $q_0 = CV$.
  - Maximum current in the circuit, $I_0 = V/R$.
  - RC is known as time constant (τ_C) of circuit.
  - $V_C + V_R = V$ ...(i)
  - Charging means q increases. So if q increase then V_C increases so from equation (i) V_R decreases and hence I decreases.



Exponential Growth

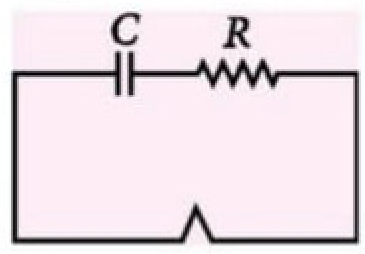


Exponential Decay

- Discharging a capacitor
  - Charge on the capacitor at any time t,

    $q = q_0 e^{-t/RC}$;

    $q_0 = CV$ = initial charge on the capacitor at $t = 0$
  - Current during discharging

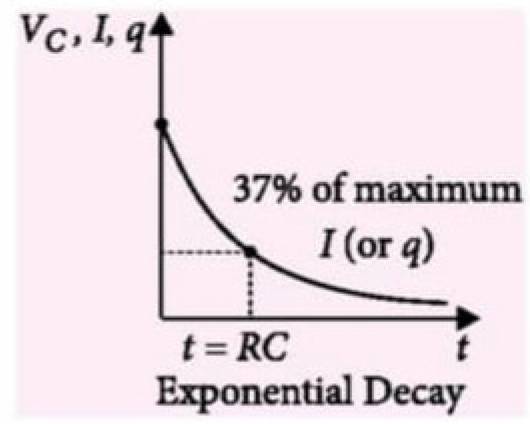


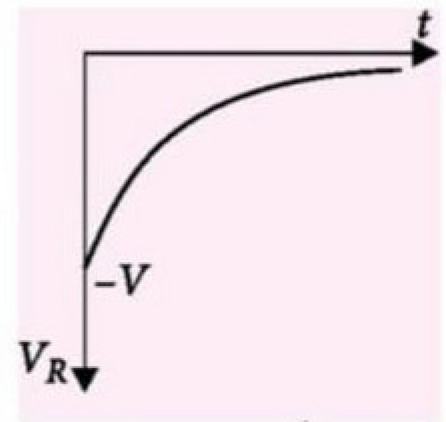
$$I = \frac{dq}{dt} = -I_0 e^{-t/RC}$$

$$I_0 = \frac{V}{R} = \text{initial current at } t = 0$$

  - $V_C + V_R = 0$

    $|V_C| = |V_R|$ ...(i)
  - Discharging means q decreases so V_C decreases. Hence from eqn (i) $|V_R|$ decreases and correspondingly I decreases.





  - In discharing all the parameters (q, V_C, V_R, I) decay exponentially.
- In both charging and discharging current decreases exponentially *i.e.* capacitor blocks D.C.
- In time $t = RC = \tau_C$ (time constant) any parameter changes by 63%.
- If time constant (τ_C) of a circuit is very less, then initial changes in any parameter (q, V_C, V_R, I) is so rapid.

WRAP it up!

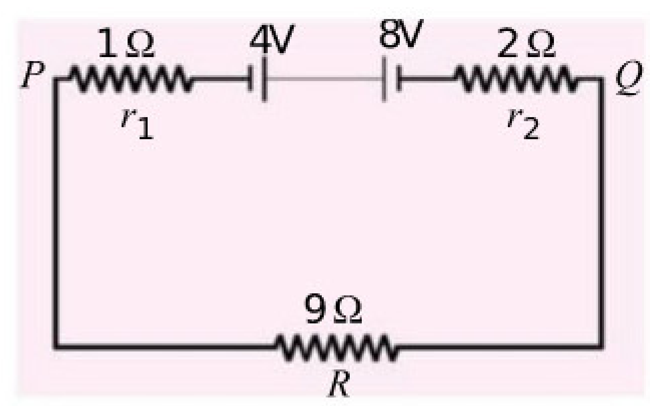
1. An electron moves in a circle of radius 10 cm with a constant speed of 4.0×10^6 m s^{-1}. The electric current at a point on the circle will be

(a) 2.0×10^{-12} A (b) 1.0×10^{-12} A
(c) 3.2×10^{-12} A (d) 6.4×10^{-12} A

2. A metal rod of length 10 cm and a rectangular cross-section of $1\text{ cm} \times \frac{1}{2}\text{ cm}$ is connected to a battery across opposite faces. The resistance will be

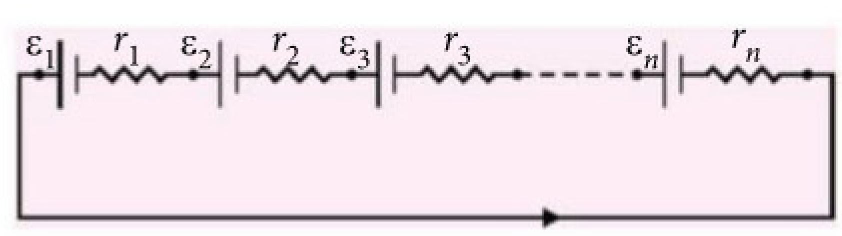
(a) maximum when the battery is connected across $1\text{ cm} \times \frac{1}{2}\text{ cm}$ faces.
(b) maximum when the battery is connected across 10 cm × 1 cm faces.
(c) maximum when the battery is connected across $10\text{ cm} \times \frac{1}{2}\text{ cm}$ faces.
(d) same irrespective of the three faces.

3. Two batteries of emf 4 V and 8 V with internal resistance 1 Ω and 2 Ω are connected in a circuit with resistance of 9 Ω as shown in figure. The current and potential difference between the points P and Q are



(a) $\frac{1}{3}$ A and 3 V (b) $\frac{1}{6}$ A and 4 V
(b) $\frac{1}{9}$ A and 9 V (d) $\frac{1}{12}$ A and 12 V

4. n batteries are connected to form a circuit as shown in the figure. The resistances denote the internal resistances of the batteries which are related to the emf's as $r_i = k\varepsilon_i$ where k is a constant with proper SI units. The solid dots represent the terminals of the batteries. Then



(a) the current through the circuit is $\frac{n}{k}$.
(b) the potential difference between the terminals of the i^{th} battery is zero.

(c) the current through the circuit is $\frac{n^2}{k}$.

(d) the potential difference between the terminals of the i^{th} battery is $\frac{\varepsilon}{k}$.

5. Two non-ideal batteries are connected in parallel. Consider the following statements :

(A) The equivalent emf is smaller than either of the two emfs.

(B) The equivalent internal resistance is smaller than either of the two internal resistances.

Which one is correct regarding the given statements?

(a) Both *A* and *B* are correct.
(b) *A* is correct but *B* is wrong.
(c) *B* is correct but *A* is wrong.
(d) Both *A* and *B* are wrong.

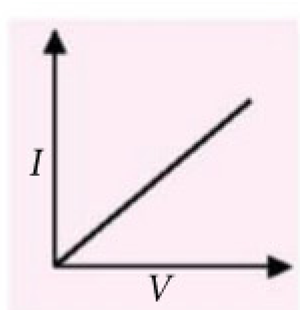
6. A metal wire of diameter 2 mm and length 100 m has a resistance of 0.5475 Ω at 20 °C and 0.805 Ω at 150 °C. Then

(a) the temperature coefficient of resistance is 3.6×10^{-3} °C^{-1}
(b) resistance at 0°C is 0.5107 Ω
(c) resistivities at 20°C is 1.72×10^{-8} Ω m
(d) All of above

7. The current in a conductor varies with time *t* as $I = 2t + 3t^2$, where *I* is in ampere and *t* in seconds. Electric charge flowing through a section of the conductor during *t* = 2 s to *t* = 3 s is

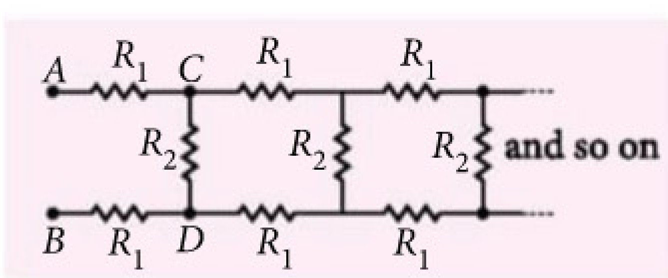
(a) 10 C (b) 24 C (c) 33 C (d) 44 C

8. *I*-*V* characteristics of a copper wire of length *L* and area of cross-section *A* is shown in figure. The slope of the curve becomes



(a) more if the experiment is performed at higher temperature
(b) more if a wire of steel of same dimension is used
(c) more if the length of the wire is increased
(d) less if the length of the wire is increased

9. A network of resistors of resistances R_1 and R_2 extends off to infinity to the right as shown in the figure.

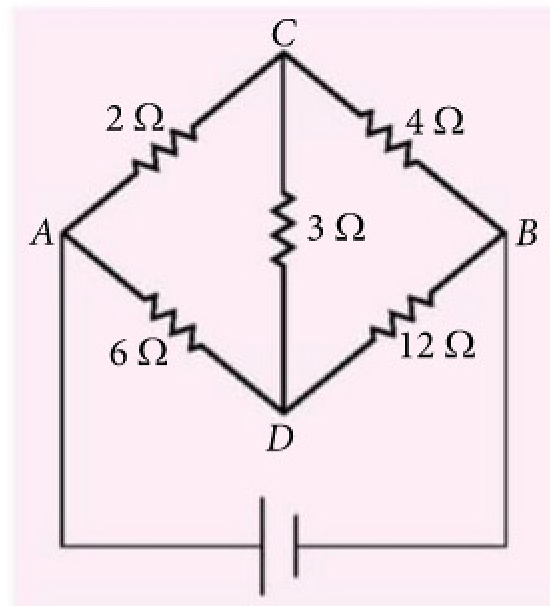


The total resistance of the network between points *A* and *B* is equal to

(a) $R_1 + \sqrt{R_1^2 + 2R_1R_2}$ (b) $R_1 - \sqrt{R_1^2 + R_1R_2}$

(c) $R_2 + \sqrt{R_2^2 + 2R_1R_2}$ (d) $R_2 - \sqrt{R_2^2 + R_1R_2}$

10. The equivalent resistance (R_{AB}) between the point *A* and *B* is

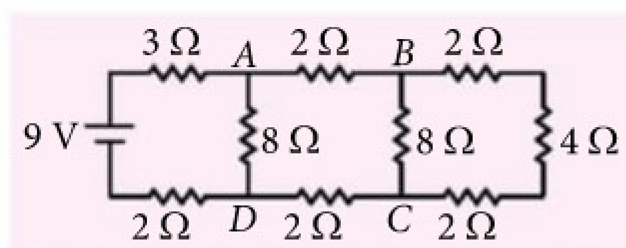


(a) 6 Ω (b) 7.5 Ω (c) 4.5 Ω (d) 8 Ω

11. A fully charged capacitor *C* discharges through a resistor *R*. After how many time constants does the stored energy drop to half its initial value?

(a) 0.20 (b) 0.02 (c) 0.35 (d) 0.16

12. Refer to the circuit shown in the figure.



The current through the

(a) 3 Ω resistor is 0.50 A
(b) 3 Ω resistor is 0.25 A
(c) 4 Ω resistor is 0.50 A
(d) 4 Ω resistor is 0.25 A

13. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be

(a) 14 A (b) 8 A (c) 10 A (d) 12 A

14. Two metal wires of identical dimensions are connected in series. If σ_1 and σ_2 are the conductivities of the metal wires respectively, the effective conductivity of the combination is

(a) $\frac{\sigma_1 + \sigma_2}{\sigma_1\sigma_2}$ (b) $\frac{\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$

(c) $\frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$ (d) $\frac{\sigma_1 + \sigma_2}{2\sigma_1\sigma_2}$

15. A potentiometer wire of length L and a resistance r are connected in series with a battery of e.m.f. E_0 and a resistance r_1. An unknown e.m.f. E is balanced at a length l of the potentiometer wire. The e.m.f. E will be given by

(a) $\frac{E_0 l}{L}$ (b) $\frac{LE_0 r}{(r+r_1)l}$

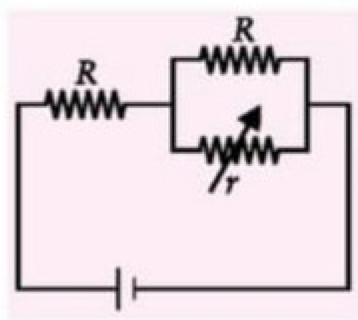
(c) $\frac{LE_0 r}{lr_1}$ (d) $\frac{E_0 r}{(r+r_1)}\cdot\frac{l}{L}$

16. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m s^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to

(a) 1.6×10^{-6} Ω m (b) 1.6×10^{-5} Ω m

(c) 1.6×10^{-8} Ω m (d) 1.6×10^{-7} Ω m

17. In the circuit shown, the resistance r is a variable resistance. If for $r = fR$, the heat generation in r is maximum then the value of f is



(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

18. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$, where a and b are positive constants. The total heat produced in R is

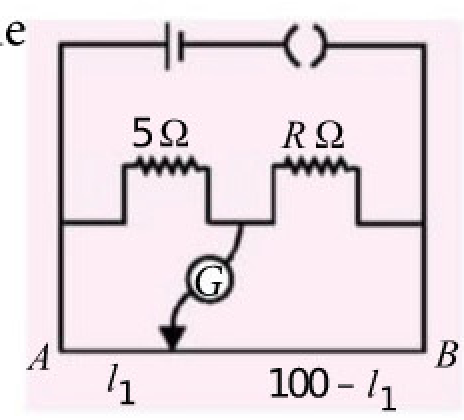
(a) $\frac{a^3R}{2b}$ (b) $\frac{a^3R}{b}$ (c) $\frac{a^3R}{6b}$ (d) $\frac{a^3R}{3b}$

19. Two cities are 150 km apart. Electric power is sent from one city to another city through copper wires. The fall of potential per km is 8 V and the average resistance per km is 0.5 Ω. The power loss in the wire is

(a) 19.2 W (b) 19.2 kW

(c) 19.2 J (d) 12.2 kW

20. The resistances in the two arms of the meter bridge are 5 Ω and R Ω respectively. When the resistance R is shunted with an equal resistance, the new balance point is at $1.6l_1$. The resistance R is



(a) 10 Ω (b) 15 Ω

(c) 20 Ω (d) 25 Ω

SOLUTIONS

1. **(b):** Consider a point A on the circle. The electron crosses this point once in every revolution. In one revolution, the electron travels $2\pi \times$ (10 cm) distance. Hence, the number of revolutions made by the electron in one second is

$$\frac{4.0\times10^6}{20\pi\times10^{-2}} = \frac{2}{\pi}\times10^7.$$

The charge crossing the point A per second is

$$\frac{2}{\pi}\times10^7\times1.6\times10^{-19}\,\text{C} = \frac{3.2}{\pi}\times10^{-12}\,\text{C}.$$

Thus, the electric current at this point is

$$\frac{3.2}{\pi}\times10^{-12}\,\text{A} \approx 1.0\times10^{-12}\,\text{A}$$

2. **(a):** Since $R = \rho\frac{l}{A}$, resistance will be maximum when l is maximum and A is least. This holds good for option (a) only.

3. **(a) :** $I = \frac{8-4}{1+2+9} = \frac{4}{12} = \frac{1}{3}\,\text{A}$

$$V_P - V_Q = 4 - \frac{1}{3}\times3 = 3\,\text{V}$$

4. **(b):** Suppose the current is I in the indicated direction. Applying Kirchoff's loop law,

$\varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2 + \varepsilon_3 - Ir_3 + ... + \varepsilon_n - Ir_n = 0$

or, $I = \frac{\varepsilon_1+\varepsilon_2+\varepsilon_3+...+\varepsilon_n}{r_1+r_2+r_3+...r_n}$

$$= \frac{\varepsilon_1+\varepsilon_2+\varepsilon_3+...+\varepsilon_n}{k(\varepsilon_1+\varepsilon_2+\varepsilon_3+...\varepsilon_n)} = \frac{1}{k}$$

The potential difference between the terminals of the i^{th} battery is

$$\varepsilon_i - Ir_i = \varepsilon_i - \left(\frac{1}{k}\right)(k\varepsilon_i) = 0$$

5. **(c)**

Monthly Test Drive CLASS XI ANSWER KEY

1.	(b)	2.	(d)	3.	(c)	4.	(c)	5.	(b)
6.	(a)	7.	(c)	8.	(c)	9.	(b)	10.	(c)
11.	(b)	12.	(c)	13.	(c)	14.	(b)	15.	(b)
16.	(a)	17.	(d)	18.	(b)	19.	(c)	20.	(a, d)
21.	(b, c)	22.	(c, d)	23.	(a,b,c)	24.	(4)	25.	(4)
26.	(7)	27.	(b)	28.	(c)	29.	(b)	30.	(a)

6. (d): Here $r = 1$ mm $= 10^{-3}$ m, $l = 100$ m,
$t_1 = 20$ °C, $R_1 = 0.5475\ \Omega$, $t_2 = 150$ °C, $R_2 = 0.805\ \Omega$
Temperature coefficient of resistance is

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{0.805 - 0.5475}{0.5475(150 - 20)} = 3.6\times10^{-3}\,°\text{C}^{-1}.$$

Resistance at 0°C,

$$R_0 = \frac{R_1}{1+\alpha t_1} = \frac{0.5475}{1+3.6\times10^{-3}\times20} = \frac{0.5475}{1.072} = 0.5107\,\Omega$$

Resistivity at 0°C,

$$\rho_0 = \frac{R_0 A}{l} = \frac{R_0\times\pi r^2}{l} = \frac{0.5107\times3.14\times(10^{-3})^2}{100}$$

$= 1.60 \times 10^{-8}\ \Omega$ m

Resistivity at 20° C,

$\rho_{20} = \rho_0\,(1 + \alpha\, t)$
$= 1.60 \times 10^{-8}\,(1 + 3.6 \times 10^{-3} \times 20)$
$= 1.60 \times 10^{-8} \times 1.072 = 1.72 \times 10^{-8}\ \Omega$ m

7. (b): Given $I = 2t + 3t^2$

As $I = \frac{dq}{dt}$ $\therefore$ $dq = Idt = (2t + 3t^2)dt$

Charge passed from $t = 2$ s to $t = 3$ s is

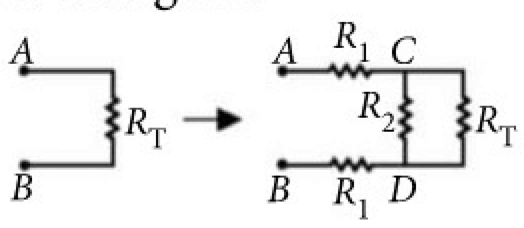
$$q = \int dq = \int_2^3 (2t + 3t^2)dt = [t^2]_2^3 + [t^3]_2^3$$

$= (9 - 4) + (27 - 8) = 24$ C

8. (d): Slope of $I - V$ graph $= \frac{\Delta I}{\Delta V} = \frac{1}{R}$

If the experiment is performed at a higher temperature, the resistance R of copper increases and hence slope decreases, so the option (a) is wrong.
In options (b) and (c), the resistances increase and so their slope become less.
In option (d), the resistance R increases and so slope decreases. Hence only option (d) is correct.

9. (a): Since the network is infinite, the resistance of the network to the right of points C and D is also equal to R_T (net resistance). We can redraw the circuit as show in figure.



Clearly, $R_T = 2R_1 + \frac{R_2 R_T}{R_2 + R_T}$ $[\because R_2 \,||\, R_T]$

or $R_T^2 - 2R_1R_T - 2R_1R_2 = 0$

or $R_T = R_1 \pm \sqrt{R_1^2 + 2R_1R_2}$

Since $R_T > 0$, $R_T = R_1 + \sqrt{R_1^2 + 2R_1R_2}$

10. (c): As $\frac{2\,\Omega}{4\,\Omega} = \frac{6\,\Omega}{12\,\Omega}$

Hence, the Wheatstone bridge is balanced.
$R_{ACB} = 2 + 4 = 6\ \Omega$, $R_{ADB} = 6 + 12 = 18\ \Omega$

$$R_{AB} = \frac{6\times18}{6+18} = 4.5\ \Omega$$

11. (c): In discharging the capacitor the charge on the capacitor varies as
$q = q_0 e^{-t/\tau}$
The energy stored in the capacitor is

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2C}e^{-2t/\tau} = U_0\,e^{-2t/\tau}$$

in which U_0 is the initial stored energy. The time at which $U = \frac{1}{2}U_0$ is found from

$$\frac{1}{2}U_0 = U_0 e^{-2t/\tau} \text{ or } 2 = e^{2t/\tau} \text{ or } t = \tau\frac{\ln 2}{2} = 0.35\tau$$

12. (d): R_{BC} (right hand side) $= \frac{8\times8}{8+8}\,\Omega$

$= 4\ \Omega$ (as $2\ \Omega + 4\ \Omega + 2\ \Omega = 8\ \Omega$)

R_{AD} (right hand side) is again $4\ \Omega$.
Equivalent resistance of the circuit,
$R = 3\ \Omega + 4\ \Omega + 2\ \Omega = 9\ \Omega$

Current drawn from battery $I = \frac{V}{R} = \frac{9}{9} = 1$ A

At A, I is equally divided ($I/2$) between 8 Ω resistance and the remaining circuit of 8 Ω. At B, ($I/2$) is equally divided ($I/4$) between the 8 Ω resistor and the remaining circuit of resistance 8 Ω. Thus, current through 4 Ω resistor is $I/4$, *i.e.*, 0.25 A.

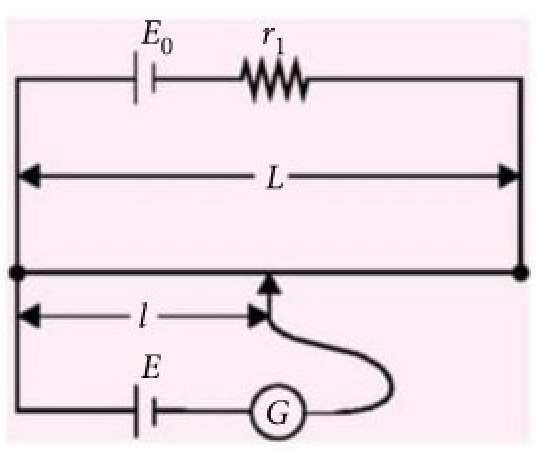
13. (d): Power of 15 bulbs of 40 W = 15 × 40 = 600 W
Power of 5 bulbs of 100 W = 5 × 100 = 500 W
Power of 5 fans of 80 W = 5 × 80 = 400 W
Power of 1 heater of 1 kW = 1000 W
∴ Total power, P = 600 + 500 + 400 + 1000 = 2500 W

When these combination of bulbs, fans and heater are connected to 220 V mains, current in the main fuse of building is given by

$$I = \frac{P}{V} = \frac{2500}{220} = 11.36\text{ A} \approx 12\text{ A}$$

14. (c)

15. (d): The current through the potentiometer wire is



$$I = \frac{E_0}{(r + r_1)}$$

and the potential difference across the wire is

$$V = Ir = \frac{E_0 r}{(r + r_1)}$$

The potential gradient along the potentiometer wire is

$$k = \frac{V}{L} = \frac{E_0 r}{(r + r_1)L}$$

As the unknown e.m.f. E is balanced against length l of the potentiometer wire,

$$\therefore \quad E = kl = \frac{E_0 r}{(r + r_1)}\frac{l}{L}$$

16. (b): $V = IR$, $I = neAv_d$ and $R = \frac{\rho l}{A}$

$$\therefore \quad V = neAv_d \times \frac{\rho l}{A} \quad \text{or} \quad \rho = \frac{V}{nev_d l}$$

Here, $V = 5$ V, $n = 8 \times 10^{28}$ m^{-3}, $v_d = 2.5 \times 10^{-4}$ m s^{-1}, $l = 0.1$ m, $e = 1.6 \times 10^{-19}$ C

∴ $\rho = 0.156 \times 10^{-4}\ \Omega\text{ m} \simeq 1.6 \times 10^{-5}\ \Omega\text{ m}$

17. (a): Let the source voltage be V.
Equivalent resistance of the circuit when $r = fR$

$$R_{eq} = R + \frac{r \times R}{r + R} = R + \frac{fR}{f+1} = \frac{(2f+1)R}{(f+1)}$$

∴ Current in the circuit, $I = \frac{V}{R_{eq}} = \frac{V(f+1)}{R(2f+1)}$

Current in the resistance $r(= fR)$

$$I_2 = \frac{I}{f+1} = \frac{V}{R(2f+1)}$$

Now, heat generated per unit time in r

$$H = I_2^2 r = \frac{V^2 f}{R(2f+1)^2}$$

For maximum H, $\frac{dH}{df} = 0$

$$\Rightarrow \frac{V^2}{R}\left[\frac{1}{(2f+1)^2} - \frac{4f}{(2f+1)^3}\right] = 0 \Rightarrow f = \frac{1}{2}$$

18. (c): Given, $Q = at - bt^2$

$$\therefore \quad I = \frac{dQ}{dt} = a - 2bt$$

At $t = 0, Q = 0 \Rightarrow I = 0$
Also, $I = 0$ at $t = a/2b$
∴ Total heat produced in resistance R,

$$H = \int_0^{a/2b} I^2 R dt = R\int_0^{a/2b}(a - 2bt)^2 dt$$

$$= R\int_0^{a/2b}(a^2 + 4b^2t^2 - 4abt)dt$$

$$= R\left[a^2 t + 4b^2\frac{t^3}{3} - 4ab\frac{t^2}{2}\right]_0^{a/2b}$$

$$= \frac{a^3 R}{b}\left[\frac{1}{2} + \frac{1}{6} - \frac{1}{2}\right] = \frac{a^3 R}{6b}$$

19. (b): Here,
Distance between two cities = 150 km
Resistance of the wire, R = (0.5 Ω km^{-1})(150 km) = 75 Ω
Voltage drop across the wire,
V = (8 V km^{-1})(150 km) = 1200 V
Power loss in the wire is

$$P = \frac{V^2}{R} = \frac{(1200\text{ V})^2}{75\ \Omega} = 19200\text{ W} = 19.2\text{ kW}$$

20. (b)

◈◈

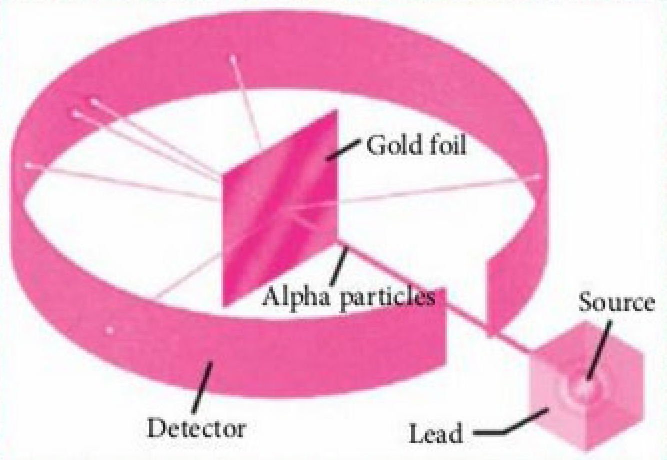
BRAIN MAP

Niels Henrik David Bohr

Niels Bohr was born in Coponhagen, Denmark, on 7th October, 1885. Bohr was a physicist who made foundational contributions to understanding atomic structure and quantum theory for which he received the Nobel Prize in Physics in 1922. Bohr developed Bohr model of atom in which he proposed that energy level of electrons are discrete and that the electrons revolve in stable orbit around the nucleus. In September 1943 he joined the British Tube Alloys nuclear weapons project. After the war he involved in establishment of CERN and became first chairman of the Nordic Institute for Theoretical Physics in 1957.

(1885-1962)

ATOMS & NUCLEI



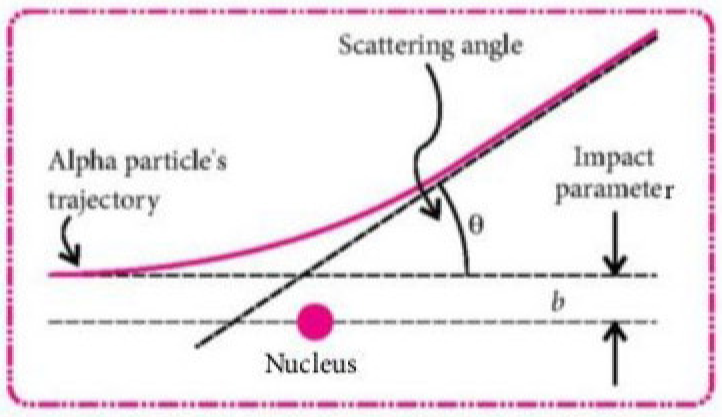
RUTHERFORD'S MODEL OF ATOM

- **Alpha particle scattering experiment**
  - The beam of α-particles is allowed to fall on a thin gold foil and gets scattered in different directions which are observed with a microscope.
  - Most of α-particles pass straight through foil and few suffer deflection through small angle and one of them gets rebound on same path by 180° deflection.
  - Shape of trajectory of scattered α-particle depends on impact parameter b as

$$b = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2 \cot\frac{\theta}{2}}{K} \quad \text{when } K = \frac{1}{2}mv^2 \rightarrow \text{K.E. of } \alpha\text{-particles.}$$

Distance of closest approach $(r_0) = \frac{1}{4\pi\varepsilon_0}\frac{4Ze^2}{mv^2}$

- **Conclusion :** An atom consists of a small and massive central core in which entire positive charge and whole mass of atom are concentrated is called nucleus and electrons revolve around it.
- **Limitation :** The electron revolving around the nucleus continuously loses its energy due to centripetal acceleration and following a spiral path, finally it should collapse into the nucleus.



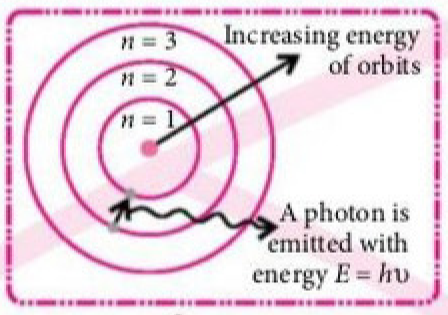
LINE SPECTRA OF HYDROGEN ATOM

- While transition between different atomic levels light is radiated in various discrete frequencies are called spectral series of hydrogen atom.
- Rydberg formula :

Wave number $\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

when R = Rydberg's constant = $1.097 \times 10^7\ \text{m}^{-1}$ and n_f and n_i are final and initial states.

- Spectral series of hydrogen atom
  - (a) Lyman series : $n_f = 1$ and $n_i = 2, 3, 4 ...$
  - (b) Balmer series : $n_f = 2$ and $n_i = 3, 4, 5 ...$
  - (c) Paschen series : $n_f = 3$ and $n_i = 4, 5, 6 ...$
  - (d) Brackett series : $n_f = 4$ and $n_i = 5, 6, 7 ...$
  - (e) Pfund series : $n_f = 5$ and $n_i = 6, 7, 8 ...$



BOHR'S ATOMIC MODEL

- **Bohr's postulates**
  - (i) An electron in an atom could revolve in certain stable orbits without the emission of radiant energy.
  - (ii) Electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $(h/2\pi)$ when

$L = mvr = \frac{nh}{2\pi}$ where h = Planck's constant.

  - (iii) An electron making a transition from its specified orbit to a lower energy orbit, a photon is emitted having energy equal to the difference between its initial and final state.

$h\upsilon = E_i - E_f$

- **Electron orbits and their energy**
  - Radius of permitted orbits

$$r_n = \frac{n^2h^2}{4\pi^2 mkZe^2} \quad \Rightarrow \quad r_n \propto n^2$$

  - Velocity of electron in n^{th} orbit

$$v_n = \frac{2\pi kZe^2}{nh} \quad \Rightarrow \quad v_n \propto \frac{1}{n}$$

  - Energy of electron in n^{th} orbit

$$E_n = \frac{-2\pi^2 mk^2 Z^2 e^4}{n^2h^2} \quad \Rightarrow \quad E_n \propto \frac{1}{n^2}$$

where the symbols have their usual meanings.

RADIOACTIVITY

- The spontaneous disintegration of the nucleus of an atom with emission of radiations is called radioactivity.
- **Law of Radioactive decay :** The rate of disintegration of nuclei at any instant is directly proportional to number of undecayed nuclei present in the sample.

$$\frac{dN}{dt} = -\lambda N(t)$$

$$\Rightarrow \ln\frac{N}{N_0} = -\lambda t$$

$$N(t) = N_0 e^{-\lambda t}$$

[N_0 = initial number of nuclei at $t = 0$; N = number of undecayed nuclei left; λ = decay constant]

- **Half life and Mean life**

Half life : The half life $T_{1/2}$ of a radionuclide is the time in which N has been reduced to one-half of its initial value.

Mean life : The mean life τ is the time at which N has been reduced to e^{-1} of its initial value.

- $T_{1/2} = \frac{\ln 2}{\lambda} = \tau \times \ln 2 = \tau \times 0.693$
- $\left(\frac{N}{N_0}\right) = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{R}{R_0}\right)$

[$R = \frac{-dN}{dt} = \lambda N$; $t/T_{1/2} = n$ = number of half lives spent in time t.]

- **Decay schemes of α, β, and γ radiations**
  - α-Decay

$${}^{A}_{Z}X \xrightarrow{\alpha\text{-decay}} {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He} + Q$$

where Q = energy released

$Q = [m_x - m_Y - m_{He}]c^2$

  - β-Decay

$${}^{A}_{Z}X \xrightarrow{\beta^+} {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \upsilon$$

$${}^{A}_{Z}X \xrightarrow{\beta^-} {}^{A}_{Z-1}Y + {}^{0}_{-1}e + \bar{\upsilon}$$

  - γ-Decay

$${}^{A}_{Z}X^* \xrightarrow{\gamma\text{-decay}} {}^{A}_{Z}X + {}^{0}_{0}\gamma - \text{Energy}$$

(Excited state) (Ground state)

COMPOSITION AND SIZE OF NUCLEUS

- Nucleus of an atom consists of protons and neutrons collectively called nucleons, and the total number of nucleons is called mass number (A).
- Mass number or atomic mass is measured in a.m.u.

$1\ \text{a.m.u} = 1.6 \times 10^{-27}\ \text{kg} = 931.5\ \text{MeVc}^{-2}$.

- Radius of a nucleus is proportional to its mass number as $R = R_0 A^{1/3}$.
- Nuclear forces are the strongest force in nature and they are short range forces which acts only upto (2-3 fm).

CONCEPT OF BINDING ENERGY

- The binding energy is defined as the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus.

$$\Delta E_b = [Zm_p + (A - Z)m_n - M_N]c^2$$

[m_p, m_n are mass of proton and neutron respectively. M_N = nuclear mass of ${}^{A}_{Z}X$]

- **Binding energy per nucleon :** The ratio of binding energy E_b of a nucleus to the mass number A.

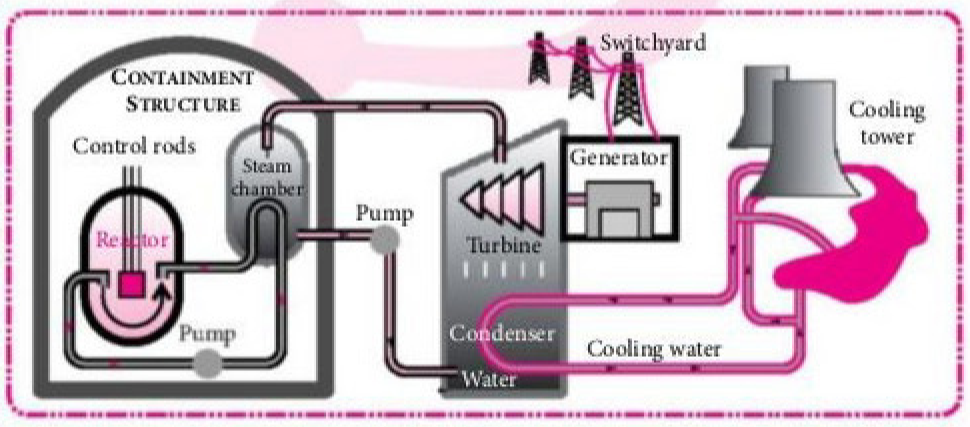
$$\therefore E_{bn} = \frac{E_b}{A}$$

NUCLEAR REACTIONS

A. **Nuclear fission :** A heavy nucleus ($A > 230$) when excited splits into two smaller nuclei of nearly comparable masses along with the emission of 3 neutrons.

$${}^{235}_{92}\text{U} + {}^{1}_{0}n \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}n + Q \quad \{Q\text{-value} = 200\ \text{MeV}\}$$

- **Application**
  - Uncontrolled chain reaction → Principle of atomic bombs.
  - Controlled chain reaction → Principle of nuclear reactors.



B. **Nuclear fusion :** Two light nuclei combine to form a single heavier nucleus is called nuclear fusion.

- Necessary condition : It can occur only under extremely high pressure and high temperature.
- Energy available per nucleon is large about 6.75 MeV
- Nuclear fusion is the source of energy in sun and stars. It occurs via two different cycles.
  - Proton-proton cycle.
  - Carbon-nitrogen cycle.

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 120

Electrostatic Potential and Capacitance

Time Taken : 60 Min.

NEET

Only One Option Correct Type

1. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at a distance of 2 cm from the centre of the sphere is

(a) zero (b) 10 V (c) 4 V (d) 10/3 V

2. The electric field strength at a distance r from the centre of a charged sphere of radius R is E. If $r > R$, how much work will be done in bringing a test charge q_0 from infinity to that point?

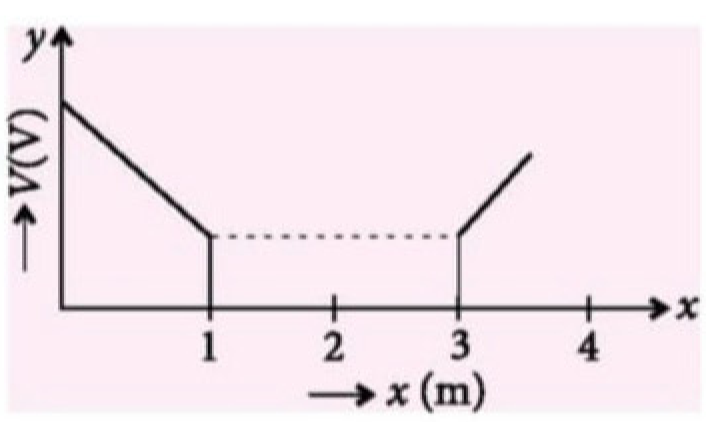
(a) q_0RE (b) $\frac{1}{2}q_0RE$

(c) q_0rE (d) $\frac{1}{2}q_0rE$

3. The plates of a parallel plate capacitor are charged to 100 V, and the battery is removed. A 2 mm thick dielectric slab is inserted between the plates. Then to maintain the same potential difference the distance between the plates is increased by 1.6 mm. The dielectric constant of the slab is

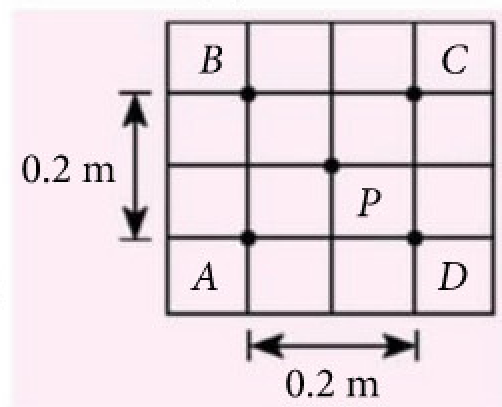
(a) 4 (b) 5 (c) 6 (d) 8

4. The variation of electric potential with distance from a fixed point is shown in the figure. What is the value of electric field at $x = 2$ m ?



(a) zero (b) 6/2 (c) 6/1 (d) 6/3

5. A, B, C, D and P are points in a uniform electric field. The potentials at these points are $V(A) = 2$ V, $V(P) = V(B) = V(D) = 5$ V and $V(C) = 8$ V. The electric field at P is



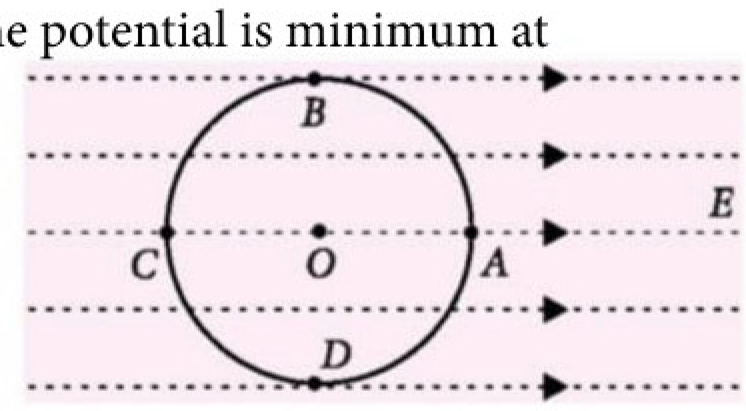
(a) 10 V m^{-1} along PB

(b) 5 V m^{-1} along PC

(c) $15\sqrt{2}$ V m^{-1} along PA

(d) 5 V m^{-1} along PA

6. The electric field in a region surrounding the origin and along the x-axis is uniform. A small circle is drawn with the center at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, and $(0, -a)$ respectively, as shown in figure then the potential is minimum at



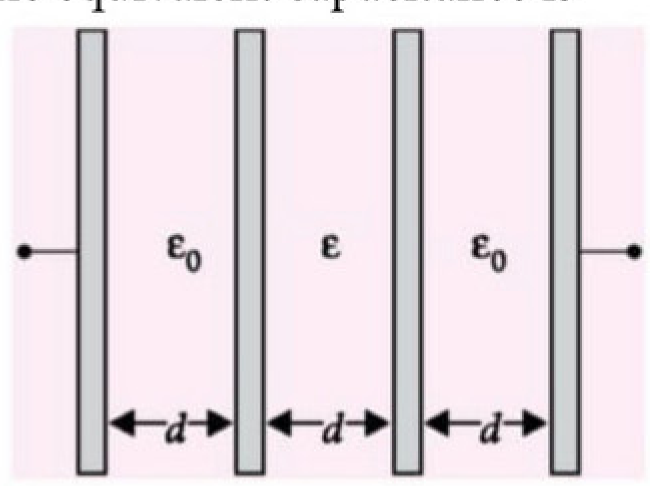
(a) A

(b) B

(c) C

(d) D

7. For the configuration of media of permittivities ε_0, ε, and ε_0 between parallel plates each of area A, as shown in figure, the equivalent capacitance is



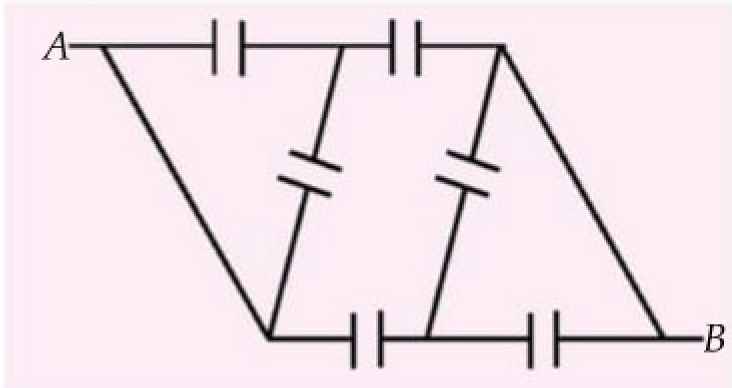
(a) $\varepsilon_0 A/d$

(b) $\varepsilon\varepsilon_0 A/d$

(c) $\frac{\varepsilon\varepsilon_0 A}{d(\varepsilon + \varepsilon_0)}$

(d) $\frac{\varepsilon\varepsilon_0 A}{(2\varepsilon + \varepsilon_0)d}$

8. A network of six identical capacitors, each of value C is made as shown in the figure. Equivalent capacitance between points A and B is

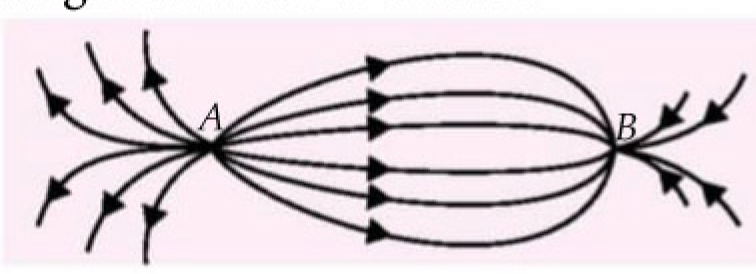


(a) $\frac{C}{4}$
(b) $\frac{3C}{4}$
(c) $\frac{4C}{3}$
(d) $3C$

9. The capacities of three capacitors are in the ratio 1 : 2 : 3. Their equivalent capacitance when connected in parallel is $\frac{60}{11}$ μF more than that when they are connected in series. The capacitance of individual capacitors in μF are

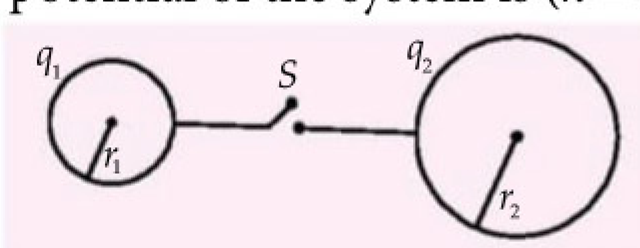
(a) 4, 6, 7 (b) 1, 2, 3 (c) 2, 3, 4 (d) 1, 3, 6

10. The spatial distribution of the electric field due to charges (A, B) is shown in figure. Which one of the following statements is correct?



(a) A is +ve and B –ve and $|A| > |B|$
(b) A is –ve and B + ve and $|A| = |B|$
(c) Both are +ve but $A > B$
(d) Both are –ve but $A > B$

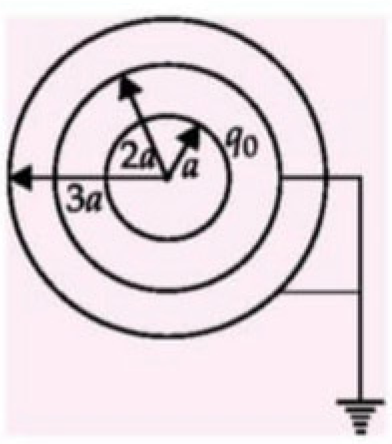
11. A system consists of two metallic spheres of radii r_1 and r_2 connected by a thin wire and a switch S as shown in figure. Initially S is open and spheres carry charges q_1 and q_2 respectively. If the switch is closed, the potential of the system is ($k = 1/4\pi\varepsilon_0$)



(a) $k\frac{q_1q_2}{r_1r_2}$ (b) $k\frac{q_1+q_2}{r_1+r_2}$

(c) $k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$ (d) $k\frac{q_1+q_2}{(r_1r_2)^{1/2}}$

12. Three concentric conducting spherical shells are arranged as shown in figure. The middle and outermost shells are earthed. The innermost sphere is given a charge q_0, what are the charges in the middle and the outermost sphere respectively?



(a) q_0 and $-q_0$ (b) $-q_0$ and zero
(c) zero and q_0 (d) $-q_0$ and q_0

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** Electron has a negative charge.
Reason : Electrons move always from a region of higher potential to a region of lower potential.

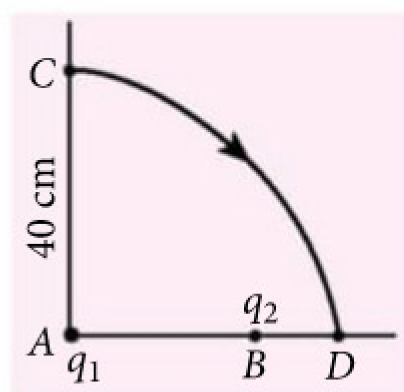
14. **Assertion :** In the absence of an externally applied electric field, the displacement per unit volume of a polar dielectric material is always zero.
Reason : In polar dielectrics, each molecule has a permanent dipole moment but these are randomly oriented in the absence of an externally applied electric field.

15. **Assertion :** A capacitor can be broken by placing large amount of charge on it.
Reason : After breakage potential, capacitor is destroyed.

JEE MAIN / ADVANCED

Only One Option Correct Type

16. Two charges q_1 and q_2 are placed 30 cm apart, as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where k is



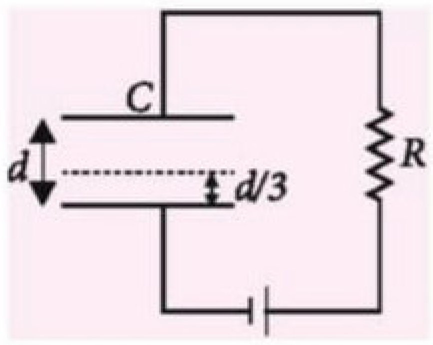
(a) $8q_2$ (b) $6q_1$ (c) $8q_1$ (d) $6q_2$

17. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is ($k = 1/4\pi\varepsilon_0$)

(a) Zero (b) $kq\left(\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right)$

(c) $2kq\left(\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right)$ (d) $k\frac{qR}{d^2}$

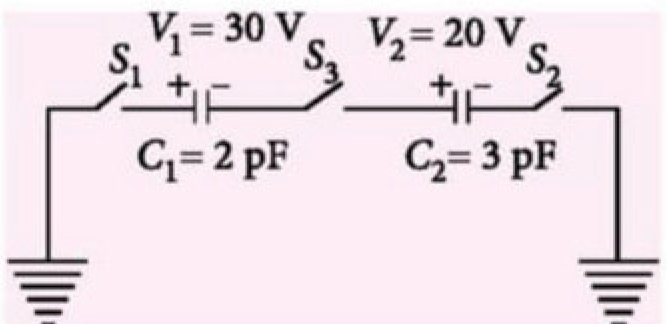
18. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is



(a) $\dfrac{6\varepsilon_0 R}{5d + 3vt}$ (b) $\dfrac{(15d + 9vt)\varepsilon_0 R}{2d^2 - 3dvt - 9v^2t^2}$

(c) $\dfrac{6\varepsilon_0 R}{5d - 3vt}$ (d) $\dfrac{(15d - 9vt)\varepsilon_0 R}{2d^2 + 3dvt - 9v^2t^2}$

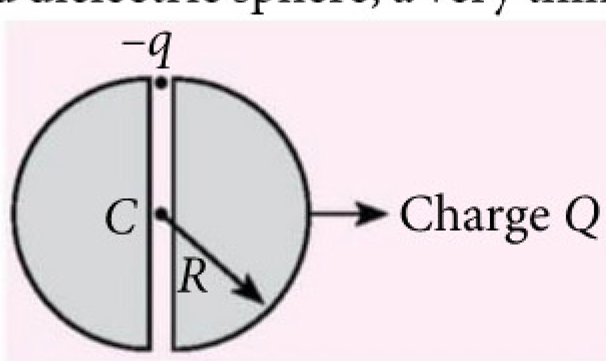
19. For the circuit shown in figure, which of the following statements is true?



(a) With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V
(b) With S_3 closed, $V_1 = V_2 = 25$ V
(c) With S_1 and S_2 closed, $V_1 = V_2 = 0$
(d) With S_1 and S_3 closed, $V_1 = 30$ V, $V_2 = 20$ V.

More than One Option Correct Type

20. In a uniformly charged dielectric sphere, a very thin tunnel has been made along the diameter as shown in figure. A charge particle $-q$ having mass m is released from rest at one end of the tunnel. For the situation described, mark the correct statement(s). (Neglect gravity.)

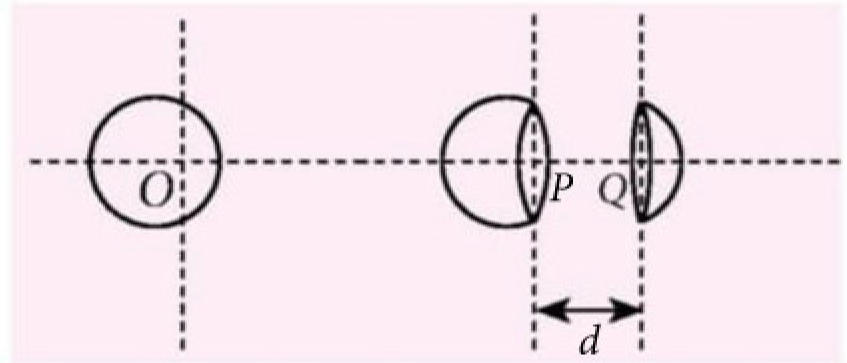


(a) Charge particle will perform SHM about centre of the sphere as mean position
(b) Time period of the particle is

$2\pi\sqrt{4\pi\varepsilon_0 m R^3/qQ}$

(c) Particle will perform oscillation but not SHM.
(d) Speed of the particle while crossing the mean position is $\sqrt{qQ/4\pi\varepsilon_0 mR}$

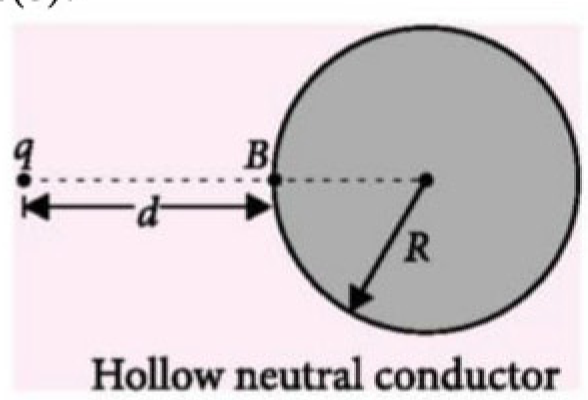
21. An insulating spherical shell of uniform surface charge density is cut into two parts and placed at a distance d apart as shown in figure. $|\vec{E}_P|$ and $|\vec{E}_Q|$ denote the electric fields at P and Q, respectively. As $d \to \infty$



(a) $|\vec{E}_P| > |\vec{E}_Q|$ (b) $|\vec{E}_P| = |\vec{E}_Q|$
(c) $|\vec{E}_P| < |\vec{E}_Q|$ (d) $|\vec{E}_P| + |\vec{E}_Q| = 0$

22. The electric potential at a point of a certain distance from a point charge is 600 V and the electric field is 200 N C^{-1}. Which of the following statements will be true?
(a) The magnitude of the charge is 20×10^{-3} C.
(b) The distance of the given point from the charge is 3 m.
(c) The potential at a distance of 9 m will be 200 V.
(d) The work done by an external agent in moving a point charge of 1 mC from the given point to a point at a distance of 9 m will be 4×10^{-4} J

23. For the situation shown in figure, mark the correct statement(s).



(a) Potential of the conductor is $q/[4\pi\varepsilon_0(d + R)]$.
(b) Potential of the conductor is $q/4\pi\varepsilon_0 d$.
(c) Potential of the conductor cannot be determined as the nature of distribution of induced charges is not known.
(d) Potential at point B due to the induced charges is $-qR/[4\pi\varepsilon_0(d + R)d]$.

Numerical Value Type

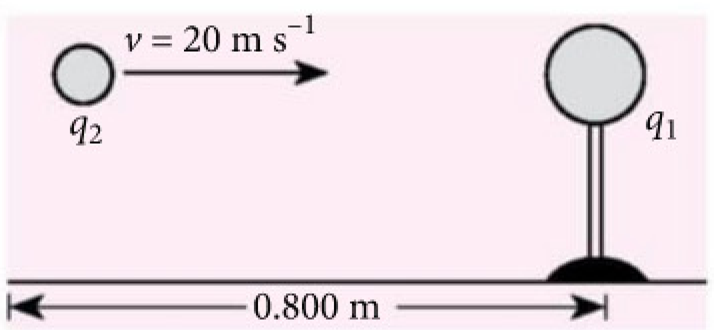
24. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = \kappa r^a$, where κ and a are constants and r is the distance from its centre. If the electric field at $r = R/2$ is 1/8 times that at $r = R$, find the value of a.

25. A parallel plate capacitor of capacity C_0 is charged to a potential V_0. E_1 is the energy stored in the capacitor when the battery is disconnected and the plate separation is doubled, and E_2 is the energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled. Find the ratio E_1/E_2.

26. The capacitance of a capacitor becomes 4/3 times its original value if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d is the separation between the plates). What is the dielectric constant of the slab?

Comprehension Type

A small metal sphere, carrying a net charge $q_1 = -2\mu C$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -8\ \mu C$ and mass 1.50 g is projected toward q_1. When the two spheres are 0.800 m apart, q_2 is moving toward q_1 with speed 20 m s^{-1} as shown in figure. Assume that the two spheres can be treated as point charges. You can ignore the force of gravity.

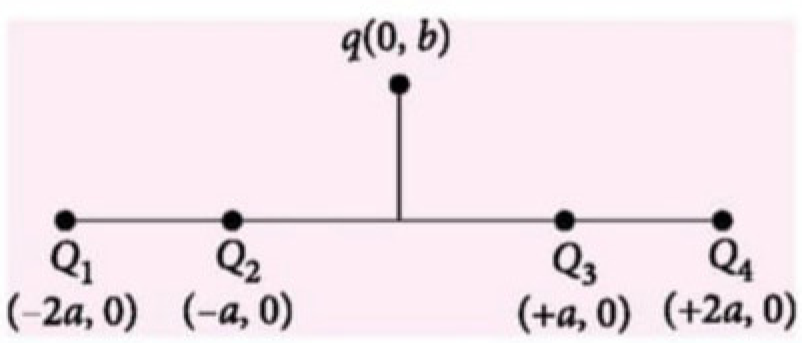


27. The speed of q_2 when the spheres are 0.400 m apart is
(a) $2\sqrt{10}$ m s^{-1} (b) $2\sqrt{6}$ m s^{-1}
(c) $4\sqrt{10}$ m s^{-1} (d) $4\sqrt{6}$ m s^{-1}

28. How close does q_2 get to q_1 ?
(a) 0.20 m (b) 0.30 m (c) 0.10 m (d) 0.15 m

Matrix Match Type

29. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x-axis at $x = -2a, -a, +a$ and $+2a$ respectively. A positive charge q is placed on the positive y-axis at a distance $b > 0$. The signs of these charges are given in column I. The direction of the forces on the charge q is given in column II. Match column I with column II

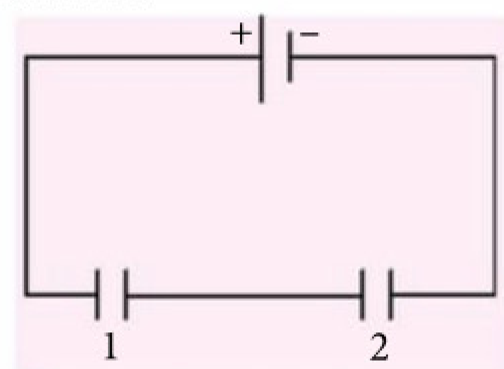


	Column-I		Column-II
A.	Q_1, Q_2, Q_3, Q_4 all positive	P.	$+x$
B.	Q_1, Q_2 positive; Q_3, Q_4 negative	Q.	$-x$
C.	Q_1, Q_4 positive; Q_2, Q_3 negative	R.	$+y$
D.	Q_1, Q_3 positive; Q_2, Q_4 negative	S.	$-y$

	A	B	C	D
(a)	R	P	S	Q
(b)	S	Q	R	P
(c)	R	P	Q	S
(d)	S	Q	P	R

30. Two identical capacitors are connected in series, and the combination is connected with a battery, as shown. Some changes in capacitor 1 are now made independently after the steady state is achieved as listed in column I. Some effects that may occur in the new steady state due to these changes on capacitor 2 are listed in column II.

Match the changes on capacitor 1 in column I with the corresponding effect on capacitor 2 in column II.



	Column-I		Column-II
A.	A dielectric slab is inserted	P.	Charge on the capacitor increases
B.	Separation between plates is increased	Q.	Charge on the capacitor decreases
C.	A metal plate is inserted connecting both plates	R.	Energy stored in the capacitor increases
D.	The left plate is grounded	S.	No change occurs

	A	B	C	D
(a)	P, R	Q	P, R	S
(b)	P	Q, R	P	S
(c)	Q, R	P	S	P
(d)	P	Q, R	S	R

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Practice questions for CBSE Exams as per the reduced syllabus, latest pattern and marking scheme issued by CBSE for the academic session 2020-21.

Series 1

CHAPTERWISE PRACTICE PAPER : Electrostatics

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory. There are 37 questions in all.
(ii) This question paper has four sections: Section A, Section B, Section C and Section D.
(iii) Section A contains twenty questions of one mark each, Section B contains seven questions of two marks each, Section C contains seven questions of three marks each, and Section D contains three questions of five marks each.
(iv) There is no overall choice. However, internal choices have been provided in two questions of one mark, two questions of two marks, one question of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.
(v) You may use the values of physical constants wherever necessary.

SECTION-A

Directions (Q. No. 1-10) : Select the most appropriate option from those given below each question.

1. Identify the wrong statement.
(a) The electrical potential energy of a system of two protons shall increase if the separation between the two is decreased.
(b) The electrical potential energy of a proton-electron system will increase if the separation between the two is decreased.
(c) The electrical potential energy of a proton-electron system will increase if the separation between the two is increased.
(d) The electrical potential energy of a system of two electrons shall increase if the separation between the two is decreased.

2. A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has a thickness $\left(\frac{3}{4}\right)d$, where d is the separation of the plates. The ratio of the capacitance C (in the presence of the dielectric) to the capacitance C_0 (in the absence of the dielectric) is

(a) $\frac{3K}{K+4}$ (b) $\frac{3}{4}K$ (c) $\frac{4K}{K+3}$ (d) $\frac{4}{3}K$.

3. If a charge on a body is 1 nC, then how many electrons are present on the body ?
(a) 6.25×10^{27} (b) 1.6×10^{19}
(c) 6.25×10^{28} (d) 6.25×10^{9}

4. Two charges of magnitude 10 units and 20 units are separated by certain distance. Now both the charges are brought to contact and again separated to initial position. What will be the ratio of initial and final force?

(a) $\frac{9}{8}$ (b) $\frac{4}{3}$ (c) $\frac{3}{2}$ (d) $\frac{8}{9}$

5. A charged particle of mass m and charge q is released from rest in an uniform electric field E. The kinetic energy of the particle after time t is

(a) $\frac{E^2q^2t^2}{2m}$ (b) $\frac{2E^2t^2}{mq}$ (c) $\frac{Eqm}{2t}$ (d) $\frac{Eq^2m}{2t^2}$

6. A neutral water molecule (dipole moment 6.0×10^{-30} C m) is placed in a uniform electric field $E = 1.5 \times 10^4$ N C^{-1} at an angle of 30°. What is the torque (in N m) acting on the water molecule ?
(a) 4.5×10^{-26} (b) 7.5×10^{-26}
(c) 9.0×10^{-26} (d) 12.0×10^{-26}

7. Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per cm length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire. The total electric flux passing through the cylindrical surface is
(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{100Q}{\varepsilon_0}$ (c) $\frac{10Q}{\pi\varepsilon_0}$ (d) $\frac{100Q}{\pi\varepsilon_0}$

8. Two parallel infinite line charges $+\lambda$ and $-\lambda$ are placed with a separation distance R in free space. The net electric field exactly mid-way between the two line charges is
(a) zero (b) $\frac{2\lambda}{\pi\varepsilon_0 R}$ (c) $\frac{\lambda}{\pi\varepsilon_0 R}$ (d) $\frac{\lambda}{2\pi\varepsilon_0 R}$

9. What is the electric potential at a distance of 9 cm from 3 nC ?
(a) 300 V (b) 270 V (c) 30 V (d) 3 V

10. The potential of the electric field produced by a point charge at any point (x, y, z) is given by $V = 3x^2 + 5$, where x, y, z are in metres and V is in volts. The intensity of the electric field at (–2, 1, 0) is
(a) $+17$ V m^{-1} (b) -17 V m^{-1}
(c) $+12$ V m^{-1} (d) -12 V m^{-1}

Directions (Q. No. 11-15) : Fill in the blanks with appropriate answer.

11. A 5 μF capacitor is connected in series with a 10 μF capacitor. When a 300 volt potential difference is applied across this combination, the total energy (in J) stored in the capacitors is ________.

12. A particle A has charge $+q$ and particle B has charge $+4q$, each of them having the same mass m. When allowed to fall from rest through the same electrical potential difference, the ratio of their speeds will become ________.

13. The path of a charged particle after it enters a region of a uniform electrostatic field with velocity perpendicular to the field will be ________.

14. The physical quantity in electrostatics analogous to temperature in heat is ________.

15. Angle between an equipotential surface and electric lines of force is ________.

OR

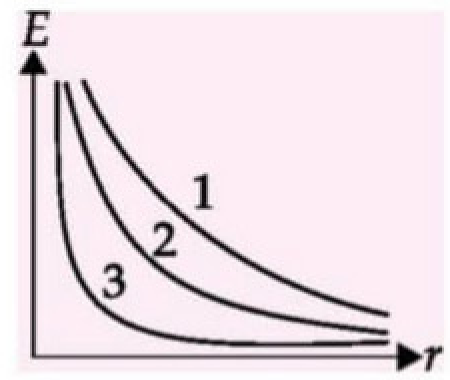
A 10 μF capacitor has been charged to a potential of 100 V. Suddenly if it explodes, the energy (in J) given out is ________.

Directions (Q. No. 16-20) : Answer the following.

16. What is the nature of symmetry of electric field due to an electric dipole?

17. Is it possible to move a charge in an electric field without doing any work? If so, how and if not, why?

18. A student plotted $E - r$ curves for a point charge, a long charged straight wire and an electric dipole but failed to label them. Identify the curves and give reason for your choice.



19. Can a charge of 1 C be given to a spherical conductor of 1 m radius? Why?

OR

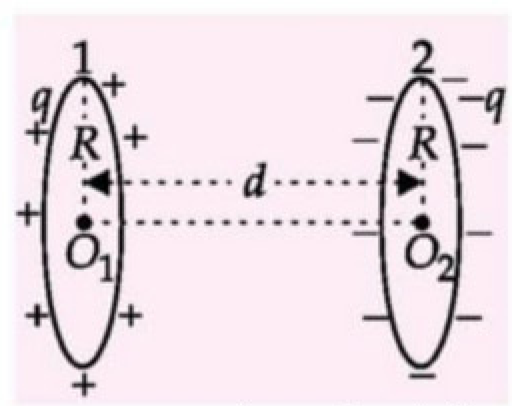
How is the number of electric field lines related to the field strength at a point?

20. Two point charges placed at a distance r apart in air exert a force F on each other. At what distance will these charges experience the same force F in a medium of dielectric constant ε_r?

SECTION-B

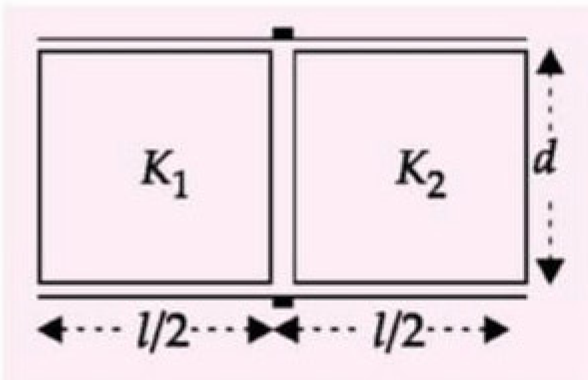
21. n tiny drops, all of same size, are given equal charges. If the drops coalesce to form a single bigger drop, then what will be the new potential of the drop? What is the surface charge density of the bigger drop?

22. Two wire rings, each having a radius R, are placed at a distance d apart with their axes coinciding as shown in the following figure.



The charges on the two rings are $+q$ and $-q$. Find the value of potential difference between the centres of the two rings.

23. Two dielectric slabs of dielectric constants K_1 and K_2 are filled in between the two plates, each of area A_1 of the parallel plate capacitor as shown in the figure. Find the net capacitance of the capacitor.



OR

A spherical capacitor has an inner sphere of radius 12 cm and outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with liquid of dielectric constant 32.

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere?

24. Plot a graph depicting variation of potential energy of an electric dipole placed in a uniform electric field with angle θ between the direction of dipole moment and field.

OR

A capacitor of capacitance C_1 is charged to a potential V. On disconnecting with the battery, it is connected with an uncharged capacitor of capacitance C_2. Find the ratio of total electrostatic potential energy before and after.

25. An electron is constrained to move along the axis of the ring of charge q and radius a. Show that the electron can perform oscillations whose frequency is given by $\omega = \sqrt{\frac{qe}{4\pi\varepsilon_0 m a^3}}$.

26. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero? Conversely, if the electric field everywhere on the surface is zero, does it imply that the charge inside is zero?

27. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

SECTION-C

28. Deduce the expression for the capacitance of a parallel plate capacitor when a dielectric slab is inserted between its plates. Assume the slab thickness is less than the plate separation.

29. Two point charges + 4e and + e are fixed at a distance a apart. Where should a third point charge q be placed on the line joining the two charges that it may be in equilibrium? In which case the equilibrium will be stable and in which unstable?

30. A small sphere of radius r and charge q is enclosed by a spherical shell of radius R and charge Q. Show that if q is positive, charge q will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge Q on the shell is.

31. Derive an expression for the torque on an electric dipole placed in a uniform electric field. Hence define dipole moment.

OR

Two point charges $+q$ and $-q$ are placed distance d apart. What are the points at which the resultant electric field is parallel to the line joining the two charges?

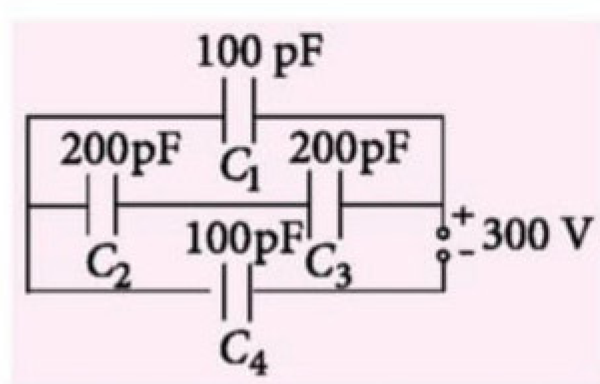
32. (a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and −2 μC(and with no external field) placed at (−9 cm, 0, 0) and (9 cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose the same system of charges is now placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^5$ N C^{-1} m^2. What would the electrostatic energy of the configuration be?

33. A charged particle, of charge 2 μC and mass 10 milligram, moving with a velocity of 1000 m s^{-1} enters a uniform electric field of strength 10^3 N C^{-1} directed perpendicular to its direction of motion. Find the velocity and displacement, of the particle after 10 s.

34. Obtain the equivalent capacitor of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor.



SECTION-D

35. Define dipole moment of an electric dipole. Show mathematically that the electric field intensity due to a short dipole at a distance d along its axis is twice the intensity at the same distance along the equatorial axis.

OR

What is an electric dipole? An electric dipole is placed in a uniform electric field at some angle θ. Deduce the expression for its potential energy. What is physical significance of electric dipole?

36. (a) Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle?

(b) Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law.

OR

Consider a system of n charges q_1, q_2 .... q_n with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ... \vec{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field $\vec{E}$ at a point P with position vector $\vec{r}_p$ due to this system of charges.

37. (a) Derive the expression for the energy stored in a parallel plate capacitor. Hence obtain the expression for the energy density of the electric field.

(b) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor.

OR

Explain, using suitable diagrams, the difference in the behaviour of a (i) conductor and (ii) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.

SOLUTIONS

1. (b) **2.** (c) **3.** (d)

4. (d) **5.** (a) **6.** (a)

7. (b) **8.** (b) **9.** (a)

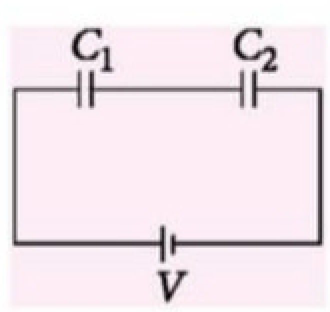
10. (c)

11. Here, $C_1 = 5\ \mu F$

$C_2 = 10\ \mu F$

$V = 300\ V$

$U = ?$



Equivalent capacitance of the circuit,

$$C = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{5 \times 10}{5 + 10} = \frac{10}{3}\ \mu F = \frac{10}{3} \times 10^{-6} F$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{10}{3} \times 10^{-6} \times (300)^2 = 0.15\ J$$

12. Mass of each charged particle = m

Let potential difference be V.

The energy of charge $+q$ when passing through potential difference V,

$$E = qV = \frac{1}{2}mv^2$$

The energy of charge $+4q$ when passing through potential difference V,

$$E' = 4q\ V = \frac{1}{2}mv'^2$$

$$\therefore \quad \frac{E}{E'} = \frac{v^2}{v'^2} = \frac{q\ V}{4qV} = \frac{1}{4} \text{ or } \frac{v}{v'} = \frac{1}{2}$$

13. After entering the region of a uniform electrostatic field with velocity perpendicular to the field, the path of the charged particle will be parabolic.

14. Potential plays the same role in electrostatics as that temperature in heat.

15. 90°

OR

$C = 10\ \mu F = 10 \times 10^{-3}$ F, $V = 100$ V

The electrostatic energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 10 \times 10^{-3} \times (100)^2 = 50\ J$$

Thus, the energy given out is 50 J.

16. The electric field due to an electric dipole exhibits cylindrical symmetry with axis as the axis of cylinder.

17. Yes, it is possible to move a charge in an electric field without doing any net work. If electric potential of initial and final points is same,

i.e., $V_A = V_B$, then $W_{AB} = q(V_B - V_A) = 0$

18. The curve 1 represents variation of electric field E with distance r normal to a long uniformly charged straight wire because here $E \propto 1/r$ and slope of curve 1 is the least. The curve 3 represents $E - r$ curve for an electric dipole because here $E \propto 1/r^3$ and slope of curve 3 is maximum.

Then, the curve 2 should represent E–r graph for a point charge, because here $E \propto 1/r^2$.

19. If a charge of 1 C is given to a spherical conductor of radius 1 m, then field at a point just near its surface,

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{9\times10^9 \times 1}{(1)^2} = 9\times10^9 \text{ V m}^{-1}$$

However, the dielectric strength of air present around the conductor is much less (about 3×10^6 V m^{-1}). So, the charge will immediately leak. Thus, it is not possible to store a charge of 1 C in a spherical conductor of radius 1 m.

OR

The number of electric field lines crossing per unit surface area around a given point is directly proportional to the strength of electric field at that point. Higher the field strength, more crowded the electric field lines around that point.

20. As, electric force between two point charges q_1 and q_2 in a medium = electric force between same charges in free space

$$\frac{k_e}{\varepsilon_r}\frac{q_1q_2}{r'^2} = k_e\frac{q_1q_2}{r^2} \text{ or } \varepsilon_r r'^2 = r^2 \text{ or } r' = \frac{r}{\sqrt{\varepsilon_r}}$$

21. Let each drop be having a radius r and charge q. Then, potential at the surface of each drop,

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

and surface density of charge, $\sigma = \frac{q}{4\pi r^2}$

When n drops coalesce to form a single bigger drop of radius R, total volume remains unchanged.

Hence, $\frac{4}{3}\pi R^3 = n\times\frac{4}{3}\pi r^3 \Rightarrow R = (n)^{1/3} r$

and total charge on the bigger drop, $Q = nq$

$\therefore$ Potential of bigger drop,

$$V' = \frac{Q}{4\pi\varepsilon_0 R} = \frac{nq}{4\pi\varepsilon_0 (n)^{1/3} r} = (n)^{2/3} V$$

and new surface charge density,

$$\sigma' = \frac{Q}{4\pi R^2} = \frac{nq}{4\pi n^{2/3} r^2} = (n)^{1/3}\sigma$$

22. Electric potential at centre O_1 of 1st ring,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[+\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

and electric potential at the centre O_2 of 2nd ring,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[+\frac{q}{\sqrt{R^2+d^2}} - \frac{q}{R}\right]$$

$\therefore$ $V_1 - V_2 =$

$$\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right) - \left(\frac{q}{\sqrt{R^2+d^2}} - \frac{q}{R}\right)\right]$$

$$= \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

23. The arrangement shown in the figure is equivalent to two capacitors each having a plate area $A/2$ and plate separation d. The two capacitors are filled with dielectrics of dielectric constants K_1 and K_2 respectively and are joined in parallel.

$\therefore$ $C_1 = \frac{K_1\varepsilon_0 A/2}{d} = \frac{K_1\varepsilon_0 A}{2d}$ and $C_2 = \frac{K_2\varepsilon_0 A}{2d}$

$\therefore$ Net capacitance of the capacitors,

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

OR

Here, $r_1 = 13$ cm, $r_2 = 12$ cm, $K = 32$, $Q = 2.5$ μC

(a) Capacitance of capacitor is

$$C = \frac{4\pi\varepsilon_0 K r_1 r_2}{r_1 - r_2} = \frac{1\times32}{9\times10^9}\times\frac{13\times10^{-2}\times12\times10^{-2}}{(13-12)\times10^{-2}}$$

or $C = 5.5 \times 10^{-9}$ F

(b) Electric potential of inner sphere is

$V_B = V_{BB} + V_{BA}$

$$= \frac{1}{4\pi\varepsilon_0 K}\left[+\frac{Q}{r_2} - \frac{Q}{r_1}\right] = \frac{Q}{4\pi\varepsilon_0 K}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

$$= \frac{9\times10^9}{32}\times2.5\times10^{-6}\left[\frac{13-12}{13\times12}\right]\times\frac{10^{-2}}{10^{-4}}$$

$= 4.5 \times 10^2$ V.

24. We know that potential energy of an electric dipole is given by

$U = -pE\cos\theta$

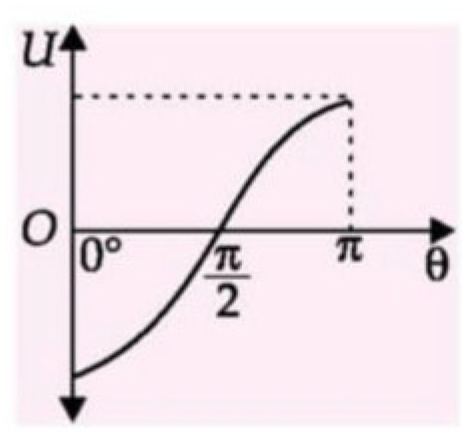
Hence,

(a) for $\theta = 0°$, $U = -pE$

(b) for $\theta = \frac{\pi}{2}$, $U = 0$

(c) for $\theta = \pi$, $U = +pE$

On the basis of this data, we plot the curve as shown in the given figure. It is a part of cosine curve.



OR

Initial value of potential energy, $U_i = \frac{1}{2}C_1V^2$

On joining C_1 with C_2, after being disconnected from the battery, common potential,

$$V' = \frac{C_1 V}{(C_1 + C_2)}$$

Hence, final value of potential energy

$$U_f = \frac{1}{2}(C_1 + C_2)V'^2$$

$$= \frac{1}{2}(C_1 + C_2)\cdot\frac{(C_1 V)^2}{(C_1 + C_2)^2} = \frac{C_1^2 V^2}{2(C_1 + C_2)}$$

$$\Rightarrow \quad \frac{U_i}{U_f} = \frac{\frac{1}{2}C_1 V^2}{\frac{1}{2}\frac{C_1^2 V^2}{(C_1 + C_2)}} = \frac{C_1 + C_2}{C_1}$$

25. Electric field at a point distant x from the ring of charge q,

$$E = k_e \frac{qx}{(a^2 + x^2)^{3/2}} = \frac{k_e qx}{a^3} \qquad \text{(for } x << a\text{)}$$

Force on the electron, $F = eE$

Acceleration of the electron,

$$\frac{d^2x}{dt^2} = \frac{F}{m} = \frac{eE}{m} = \frac{k_e qxe}{ma^3}$$

As $\frac{d^2x}{dt^2} = \omega^2 x, \omega = \sqrt{\frac{d^2x/dt^2}{x}} = \sqrt{\frac{k_e qxe/ma^3}{x}}$

or $\omega = \sqrt{\frac{qe}{4\pi\varepsilon_0 ma^3}}$

26. As $q = 0$, so from the Gauss's theorem, we have

$$\phi_E = \oint \vec{E}\,.\,d\vec{S} = 0$$

Clearly, the above equation does not imply that $\vec{E}$ is necessarily zero at all points on the Gaussian surface. It may also be possible that $\vec{E}$ is non-zero but it is perpendicular to $d\vec{S}$ at all points on the surface, even then the integral

$\oint \vec{E}\,.\,d\vec{S}$ would be zero.

However, the converse is true. If $\vec{E}$ is zero at every point on Gaussian surface, then from Gauss's theorem

$\phi_E = \oint \vec{E}\,.\,d\vec{S} = \frac{q}{\varepsilon_0}$, we get, $q = 0$

i.e., no net charge is enclosed by the Gaussian surface.

27. Here, $C_1 = 600$ pF, $V_1 = 200$ V, $C_2 = 600$ pF, $V_2 = 0$

On connecting charged capacitor to uncharged capacitor, the common potential V across the capacitors is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{600\times10^{-12}\times200 + 600\times10^{-12}\times0}{(600+600)\times10^{-12}}$$

or $V = 100$ V

Energy stored in capacitors before connection is

$$U_i = \frac{1}{2}C_1V_1^2 + 0 = \frac{1}{2}\times600\times10^{-12}\times200^2$$

or $U_i = 12\ \mu$J

Energy stored in capacitors after connection is

$$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(600+600)\times10^{-12}\times100^2$$

or $U_f = 6\ \mu$J

Hence the energy lost in the process is

$\Delta U = U_i - U_f = (12 - 6)\ \mu$J or $\Delta U = 6\ \mu$J.

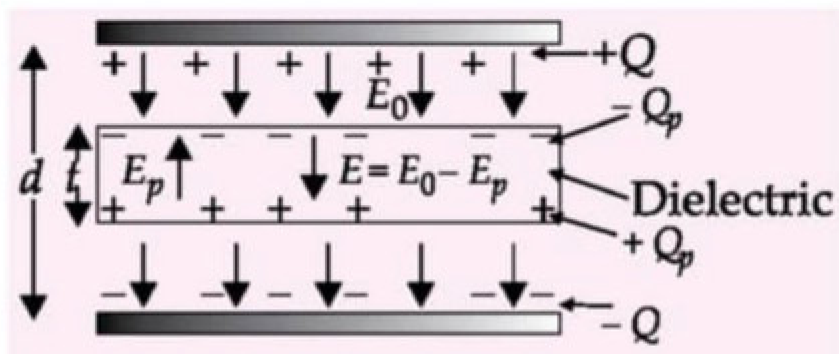
28. The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum between its plates is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

When a dielectric slab of thickness $t < d$ is placed between the plates, the field E_0 polarises the dielectric.



A dielectric slab placed in a parallel plate capacitor

This induces charge $-Q_p$ on the upper surface and $+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of $\vec{E}_0$. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}\left[\because\ \sigma_p = \frac{Q}{A} = P, \text{polarisation density}\right]$$

The net electric field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{K} \quad \left[\because\ \frac{E_0}{E_0 - E_p} = K\right]$$

where K is the dielectric constant of the slab. So between the capacitor plates, the field E exists over a distance t and field E_0 exists over the remaining distance $(d - t)$. Hence the potential difference between the capacitor plates is

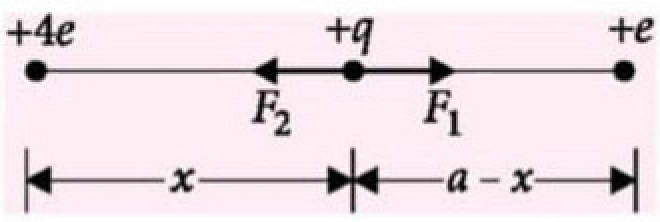
$$V = E_0(d-t) + Et = E_0(d-t) + \frac{E_0}{K}t \quad \left[\because \frac{E_0}{E} = K\right]$$

$$= E_0\left(d - t + \frac{t}{K}\right) = \frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)$$

The capacitance of the capacitor on introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}.$$

29. Suppose the three charges are placed as shown in the figure. Let the charge q be positive.



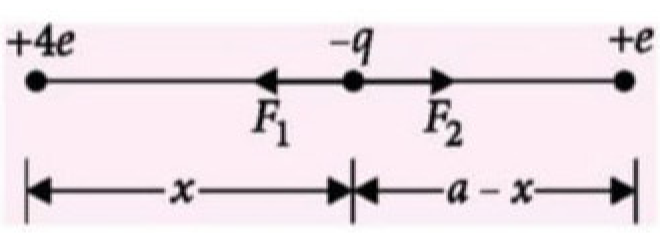
For the equilibrium of charge $+q$, we must have,
Force of repulsion F_1 between + $4e$ and $+q$
= Force of repulsion F_2 between + e and $+q$

or $\quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{4e \times q}{x^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e \times q}{(a-x)^2}$

or $\quad 4(a-x)^2 = x^2$ or $2(a-x) = \pm x$

$$x = \frac{2a}{3} \text{ or } 2a.$$

As the charge q is placed between + $4e$ and $+e$, so only $x = 2a/3$ is possible. Hence for equilibrium, the charge q must be placed at a distance $2a/3$ from the charge + $4e$.
We have considered the charge q to be positive. If we displace it slightly towards charge e, from the equilibrium position, then F_1 will decrease and F_2 will increase and a net force $(F_2 - F_1)$will act on q towards left *i.e.*, towards the equilibrium position. Hence the equilibrium of positive q is stable.
Now if we take charge q to be negative, the force F_1 and F_2 will be attractive, as shown in figure.



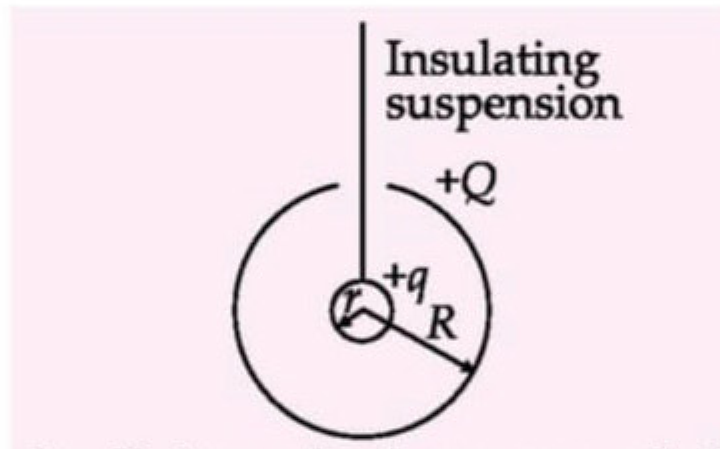
The charge $-q$ will still be in equilibrium at $x = 2a/3$. However, if we displace charge $-q$ slightly towards right, then F_1 will decrease and F_2 will increase. A net force $(F_2 - F_1)$ will act on $-q$ towards right *i.e.*, away from the equilibrium position. So the equilibrium of the negative q will be unstable.

30. Consider a small sphere of radius r placed inside a large spherical shell of radius R. Let the spheres carry charges q and Q, respectively.
Total potential on the outer sphere,
V_R = Potential due to its own charge Q
+ potential due to the charge q on the inner sphere

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} + \frac{q}{R}\right]$$

Potential on the inner sphere due to its own charge is

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$



Small charged sphere suspended inside a charged spherical shell.

As the potential at every point inside a charged sphere is the same as that on its surface, so potential on the inner sphere due to charge Q on outer sphere is

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

$\therefore \quad$ Total potential on inner sphere

$$V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{Q}{R}\right]$$

Hence the potential difference is

$$V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$$

So if q is positive, the potential of the inner sphere will always be higher than that of the outer sphere. Now if the two spheres are connected by a conducting wire, the charge q will flow to the outer sphere, irrespective of the charge Q already present on the outer sphere. In fact this is true for conductors of any shape.

31. Consider an electric dipole consisting of charges $+q$ and $-q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with it. It has a dipole moment of magnitude, $p = q \times 2a$
Force exerted on charge + q by field $\vec{E} = q\vec{E}$
(along $\vec{E}$)

Force exerted on charge $-q$ by field $\vec{E} = -q\vec{E}$ (opposite to $\vec{E}$)

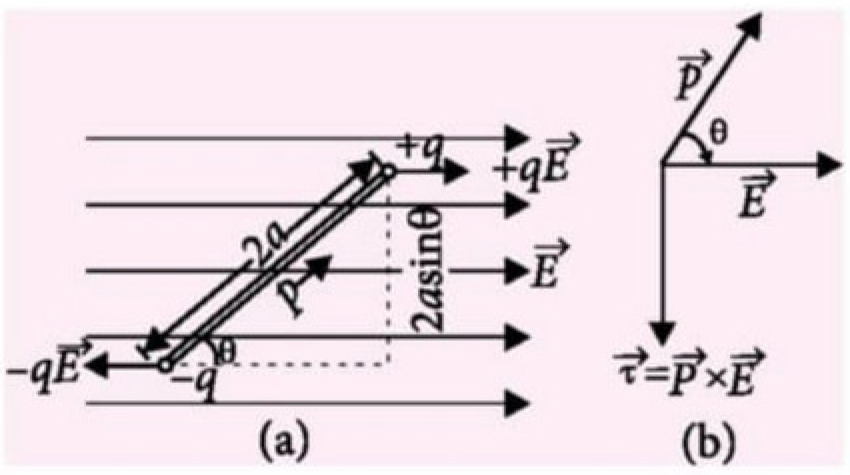
$\vec{F}_{net} = +q\vec{E} - q\vec{E} = 0.$

Hence the net translating force on a dipole in a uniform electric field is zero. But the two equal and opposite forces act at different points of the dipole. They form a couple which exerts a torque.

Torque = Either force × Perpendicular distance between the two forces

$$\tau = qE \times 2a\sin\theta = (q \times 2a)E\sin\theta$$

or $\tau = pE\sin\theta$ $(p = 2aq)$



As the direction of torque $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, so we can write

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The direction of vector $\vec{\tau}$ is that in which a right handed screw would advance when rotated from $\vec{p}$ to $\vec{E}$. As shown in figure, the direction of vector $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, and points into the plane of paper.

When the dipole is released, the torque $\vec{\tau}$ tends to align the dipole with the field $\vec{E}$ *i.e.*, tends to reduce angle θ to 0. When the dipole gets aligned with $\vec{E}$, the torque $\vec{\tau}$ becomes zero.

Clearly, the torque on the dipole will be maximum when the dipole is held perpendicular to $\vec{E}$. Thus

$$\tau_{max} = pE\sin 90° = pE.$$

Dipole moment : As $\tau = pE\sin\theta$

If $E = 1$ unit, $\theta = 90°$, then $\tau = p$

Hence dipole moment may be defined as the torque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.

OR

As shown in figure, suppose the charges $+q$ and $-q$ are located at points A and B distance d apart.

Let P be a point such that $AP = r_1$ and $BP = r_2$.

Electric field at P due to charge $+q$ is

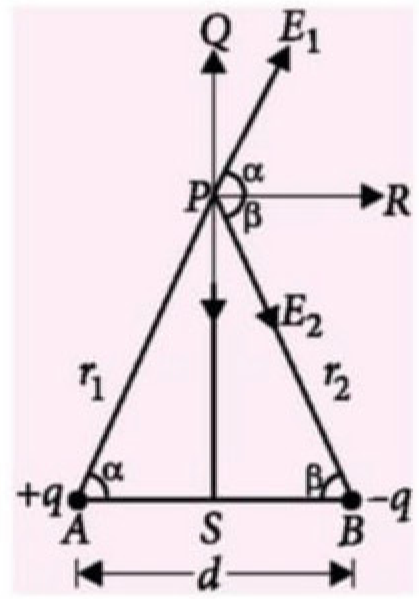
$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1^2},$$

along AP produced.

Electric field at P due to charge $-q$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_2^2},$$

along PB produced.



Draw $PR||AB$ and PQ, $PS \perp PR$. If α and β are the angles made by AP and BP with AB, then

Component of E_1 along $PR = E_1\cos\alpha$

Component of E_1 along $PQ = E_1\sin\alpha$

Component of E_2 along $PR = E_2\cos\beta$

Component of E_2 along $PS = E_2\sin\beta$

The resultant field will be parallel to PR if the components $E_1\sin\alpha$ and $E_2\sin\beta$ are equal and opposite or zero. Hence

(i) When $E_1\sin\alpha = E_2\sin\beta$

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r_1^2}\cdot\frac{PS}{r_1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_2^2}\times\frac{PS}{r_2} \quad \text{or} \quad r_1 = r_2$$

(ii) When $E_1\sin\alpha = E_2\sin\beta = 0$, we have $\sin\alpha = \sin\beta = 0$

i.e., $\alpha = 0°$ or $180°$ and $\beta = 0°$ or $180°$

Thus the resultant intensity will be parallel to the line joining A and B if (i) P lies on the perpendicular bisector of AB, or (ii) P lies on either side of AB *i.e.*, $\alpha = \beta = 0°$ or $180°$

32. (a) $q_1 = 7\ \mu C = 7 \times 10^{-6}$ C, $q_2 = -2 \times 10^{-6}$ C, $r = 18$ cm $= 0.18$ m

Electrostatic potential energy of the two charge system is

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r}$$

$$= \frac{9\times10^9\times7\times10^{-6}\times(-2)\times10^{-6}}{0.18} = -0.7 \text{ J}$$

(b) Work required to separate two charges infinitely away from each other,

$W = U_2 - U_1 = 0 - U = -(-0.7) = 0.7$ J

(c) Energy of the two charges in the external electric field = Energy of interaction of two charges with the external electric field + Mutual interaction energy of the two charges

$$= q_1V(r_1) + q_2V(r_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$= q_1\frac{A}{r_1} + q_2\frac{A}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \quad \left[\because\ V = Er = \frac{A}{r}\right]$$

$$= \left[\frac{7\,\mu C}{0.09\text{ m}} + \frac{-2\,\mu C}{0.09\text{ m}}\right] \times 9\times10^5\text{ NC}^{-1}\text{m}^2 - 0.7\text{ J}$$

= (70 – 20) – 0.7 = 50 – 0.7 = 49.3 J

33. The velocity of the particle, normal to the direction of field

v_x = 1000 m s^{-1}, is constant.

The velocity of the particle, along the direction of field, after 10 s, is given by

$v_y = u_y + a_y t$

$$= 0 + \frac{qE_y}{m}t = \frac{2\times10^{-6}\times10^3\times10}{10\times10^{-6}} = 2000\text{ m s}^{-1}$$

The net velocity after 10 s,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1000)^2 + (2000)^2} = 1000\sqrt{5}\text{ m s}^{-1}$$

Displacement, along the x-axis, after 10 s,

x = 1000 × 10 m = 10000 m

Displacement along y-axis (in the direction of field) after 10 s,

$$y = u_y t + \frac{1}{2}a_y t^2 = (0)t + \frac{1}{2}\frac{qE_y}{m}t^2$$

$$= \frac{1}{2}\times\frac{2\times10^{-6}\times10^3}{10\times10^{-6}}\times(10)^2 = 10000\text{ m}$$

Net displacement,

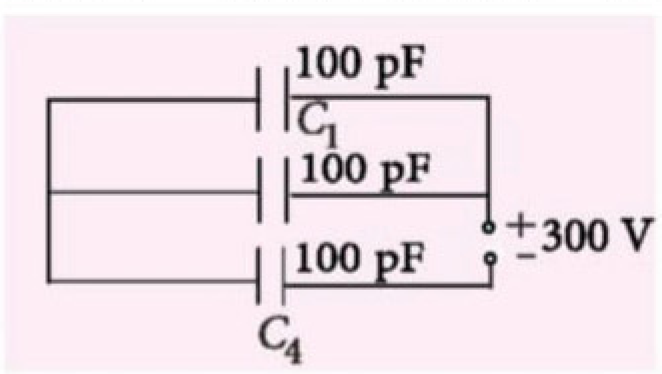
$$r = \sqrt{x^2 + y^2} = \sqrt{(10000)^2 + (10000)^2} = 10000\sqrt{2}\text{ m.}$$

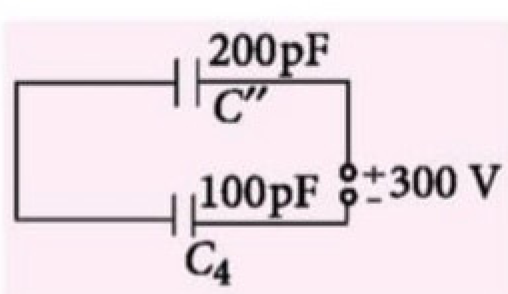
34. Since C_2 and C_3 are in series so

$$C' = \frac{C_2\times C_3}{C_2 + C_3} = 100\text{ pF}$$

Now, C_1 and C' are in parallel so

$C'' = C_1 + C' = 100 + 100$ or $C'' = 200$ pF





Since C_4 and C'' are in series, so net capacitance of the network is

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} = \frac{1+2}{200}$$

or $C = \frac{200}{3}$ pF = 66.7 pF

Net charge stored on the combination is

$$Q = CV = \frac{200}{3}\times10^{-12}\times300 = 2\times10^{-8}\text{ C}$$

As C'' and C_4 are in series, so

$Q'' = Q_4 = Q$

or $Q'' = Q_4 = 2\times10^{-8}$ C

and hence $V'' = \frac{Q''}{C''} = \frac{2\times10^{-8}\text{C}}{200\times10^{-12}\text{F}} = 100$ V

and $V_4 = \frac{Q_4}{C_4} = \frac{2\times10^{-8}\text{C}}{100\times10^{-12}\text{F}} = 200$ V

C_1 and C' are in parallel, so

$V_1 = V' = V''$

or $V_1 = V' = 100$ V

Hence, $Q_1 = C_1V_1 = 100\times10^{-12}\times100 = 1\times10^{-8}$ C

and $Q' = C'V' = 100\times10^{-12}\times100 = 1\times10^{-8}$ C

C_2 and C_3 are in series, so

$Q_2 = Q_3 = Q' = 1\times10^{-8}$ C

Hence, $V_2 = \frac{Q_2}{C_2} = \frac{1\times10^{-8}\text{C}}{200\times10^{-12}\text{F}} = 50$ V

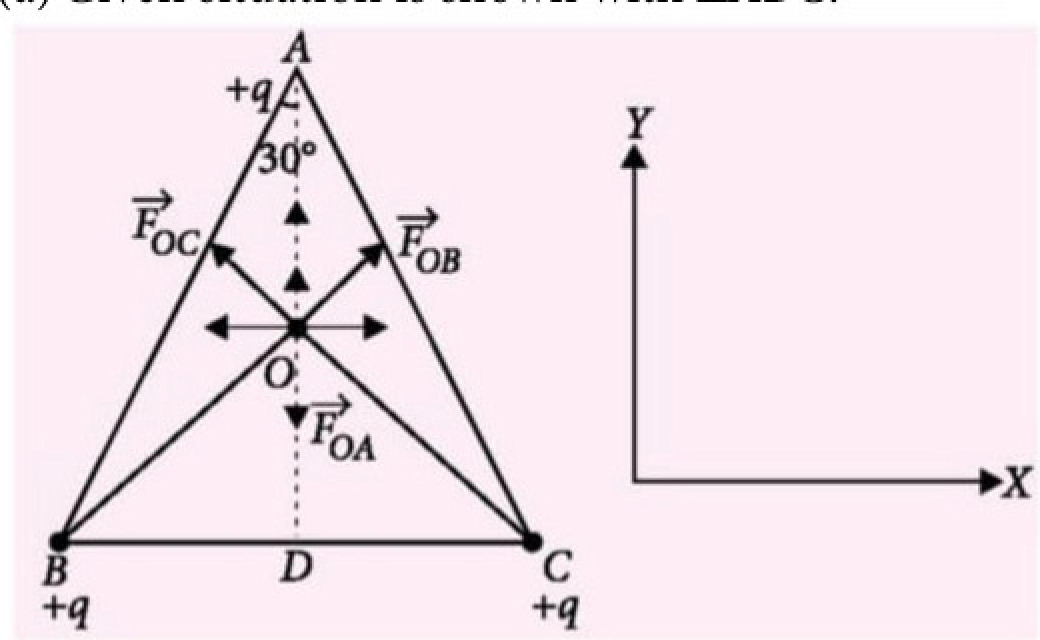
and $V_3 = \frac{Q_3}{C_3} = \frac{1\times10^{-8}\text{C}}{200\times10^{-12}\text{F}} = 50$ V.

35. *Refer to answer 25 and 22, Page no. 13* (MTG CBSE Champion Physics Class 12).

OR

Refer to answer 39, Page no. 37 (MTG CBSE Champion Physics Class 12).

36. (a) Given situation is shown with ΔABC.



In Δ ABD, $\cos 30° = \frac{AD}{AB} = \frac{AD}{l}$

or $AD = l\cos 30° = \frac{\sqrt{3}l}{2}$

As $AO = \frac{2}{3}AD = \frac{2}{3}\times\frac{\sqrt{3}l}{2}$ or $AO = \frac{l}{\sqrt{3}}$ ....(i)

Similarly $BO = CO = \frac{l}{\sqrt{3}}$

Forces on charge $+Q$ at O due to charges $+q$ at A, B and C are

$$F_{OA} = F_{OB} = F_{OC} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{\left(\frac{l}{\sqrt{3}}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{3Qq}{l^2}$$

Horizontal component of net force on $+Q$ charge at O is

$F_x = F_{OB}\cos 30° - F_{OC}\cos 30° = 0$

or $F_x = 0$ ...(ii)

and vertical component of net force on $+Q$ charge at O is

$F_y = F_{OB}\sin 30° + F_{OC}\sin 30° - F_{OA}$

or $F_y = F_{OA}\,[2\sin 30° - 1]$ $[\because F_{OB} = F_{OC} = F_{OA}]$

or $F_y = F_{OA}\left[2\times\frac{1}{2} - 1\right] = F_{OA}\,[1-1]$

or $F_y = 0$ ...(iii)

So, net force on charge $+Q$ at O is

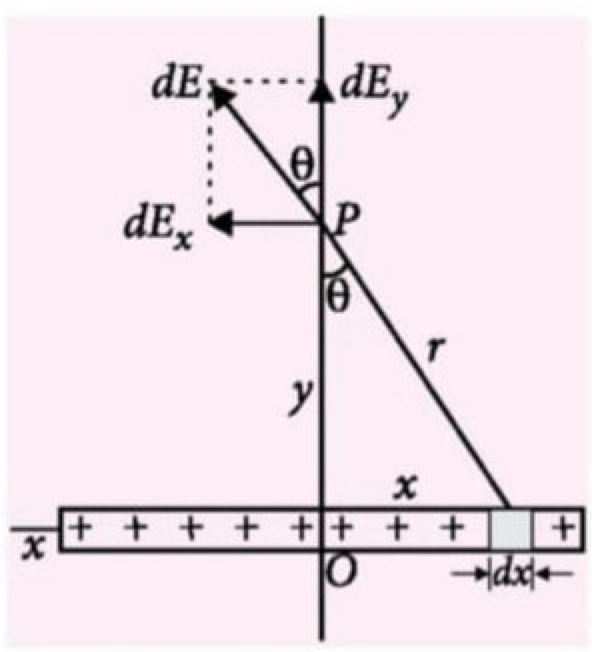
$F_{\text{net}} = \sqrt{F_x^2 + F_y^2}$ or $F_{\text{net}} = 0$

(b) Consider an infinite line of charge with uniform linear charge density λ, as shown in figure. We wish to calculate its electric field at any point P at a distance y from it. The charge on small element dx of the line charge will be

$dq = \lambda dx$

The electric field at the point P due to the charge element dq will be

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dx}{y^2 + x^2}$$



The field dE has two components :

$dE_x = -dE\sin\theta$ and $dE_y = dE\cos\theta$

The negative sign in x-component indicates that $d\vec{E}_x$ acts in the negative x-direction. Every charge element on the right has a corresponding charge element on the left. The x-components of two such charge elements will be equal and opposite and hence cancel out. The resultant field $\vec{E}$ gets contributions only from y-components and is given by

$$E = E_y = \int dE_y = \int_{x=-\infty}^{x=+\infty} dE\cos\theta$$

$$= 2\int_{x=0}^{x=\infty}\frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dx}{y^2+x^2}\cos\theta = \frac{\lambda}{2\pi\varepsilon_0}\int_{x=0}^{x=\infty}\frac{dx}{y^2+x^2}\cos\theta$$

Now $x = y\tan\theta$

$dx = y\sec^2\theta\, d\theta$

$$\therefore \quad E = \frac{\lambda}{2\pi\varepsilon_0}\int_{\theta=0}^{\theta=\pi/2}\cos\theta\frac{y\sec^2\theta d\theta}{y^2(1+\tan^2\theta)}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\int_{\theta=0}^{\theta=\pi/2}\cos\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 y}[\sin\theta]_0^{\pi/2}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\left(\sin\frac{\pi}{2} - \sin 0\right);\ E = \frac{\lambda}{2\pi\varepsilon_0 y}$$

OR

Refer to answer 8, Page no. 11 (MTG CBSE Champion Physics Class 12).

37. *Refer to answer 78, Page no. 44* (MTG CBSE Champion Physics Class 12).

OR

Refer to answer 44, Page no. 37 (MTG CBSE Champion Physics Class 12).

CLASS-XI

Brush Up for NEET/JEE 2021

Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 2 Kinematics

MOTION

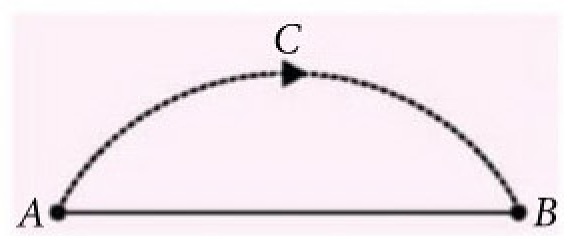
- Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer.
- If a body changes its position with time, it is said to be moving else it is at rest. Motion is always relative to the observer.
- To locate the position of a particle we need a reference frame. A commonly used reference frame is cartesian coordinate system or x-y-z coordinate system.
  - The coordinates (x, y, z) of the particle specify the position of the particle with respect to origin of that frame.
  - If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest with respect to this frame.
  - The reference frame is chosen according to problems.
  - If frame is not mention, then ground is taken as reference frame.
- If only one coordinate changes with time, motion is one dimensional motion (1 - D) or rectilinear motion.

  If only two coordinates change with time, motion is two dimensional (2 - D) or motion in a plane.

  If all three coordinates change with time, motion is three dimensional (3 - D) or motion in space.

DISTANCE AND DISPLACEMENT

- Total length of path covered by the particle, in definite time interval is called distance. The length of path ACB is called the distance travelled by the body.



- Overall, body is displaced from A to B. A vector for A to B *i.e.*, $\overrightarrow{AB}$ is its displacement vector or displacement that is the minimum distance. Displacement is directed from initial position to final position.
- For a moving body, distance can not have zero or negative values but displacement may be +ve, –ve or zero.
- If motion is in straight line without change in direction, then

  distance = |displacement|

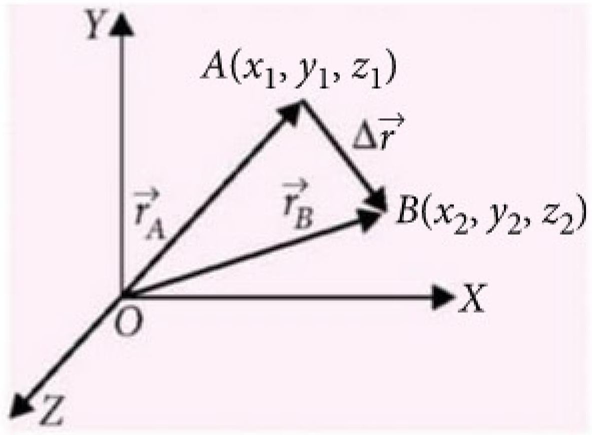
  If motion is not in straight line, then

  distance > |displacement|
- Magnitude of displacement may be equal or less than distance but never greater than distance.

  distance ≥ |displacement|

- Displacement in terms of position vector :

Let a body is displaced from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ then its displacement is given by vector $\overrightarrow{AB}$.



From ΔOAB

$$\vec{r}_A + \overrightarrow{AB} = \vec{r}_B \quad \text{or} \quad \overrightarrow{AB} = \vec{r}_B - \vec{r}_A$$

$$\because \ \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \ \text{ and } \ \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\therefore \ \overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

or $\overrightarrow{AB} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

SPEED AND VELOCITY

- Speed is related to distance and it is a scalar while velocity is related to displacement and it is a vector.
- For a moving body speed can not have zero or negative values but velocity can have.
- **Average speed**
  - It is defined for a time interval. Average speed of a trip

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

  - Let Δs be the distance travelled in the time interval t to $t + \Delta t$. The average speed in this time interval is

$$v_{av} = \frac{\Delta s}{\Delta t}$$

  - If a particle travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively, then total distance travelled $s = s_1 + s_2 + s_3 + ... + s_n$

Total time taken $= \dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dfrac{s_3}{v_3} + ... + \dfrac{s_n}{v_n}$

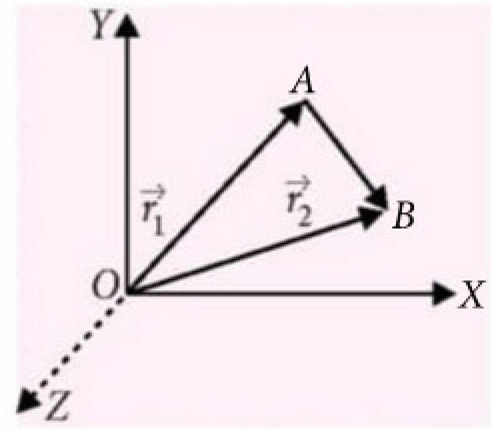
Average speed of a trip $= \dfrac{s_1 + s_2 + s_3 + ... + s_n}{\left(\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + ... + \dfrac{s_n}{v_n}\right)}$

- **Instantaneous speed**
  - The speed at a particular instant is defined as instantaneous speed (or speed).
  - If Δt approaches zero, average speed becomes instantaneous speed.

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

i.e., instantaneous speed is the time derivative of distance.

- **Average velocity**
  - Velocity is the rate of change of position vector or change in position per unit time.
  - Suppose a particle is at a point A at time t_1 and B at time t_2. Position vectors of A and B are $\vec{r}_1$ and $\vec{r}_2$. The displacement in this time interval is the vector $\overrightarrow{AB} = (\vec{r}_2 - \vec{r}_1)$. The average velocity in this time interval is,



$$\vec{v}_{av} = \frac{\text{displacement}}{\text{time interval}}$$

$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

Here, $\overrightarrow{AB} = \vec{r}_2 - \vec{r}_1$ = change in position vector.

  - For small time interval between t and $t + \Delta t$, change in position vector is Δr then average velocity in Δt time interval is, $v_{av} = \dfrac{\Delta r}{\Delta t}$
- **Instantaneous velocity**
  - It is the velocity at a particular instant. If time interval approaches zero then average velocity become instantaneous velocity.

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

i.e., instantaneous velocity is the time derivative of position vector.

  - Magnitude of instantaneous velocity is the instantaneous speed.
  - When a particle moves with constant velocity, its average velocity, its instantaneous velocity and its speed all are equal.

ACCELERATION

- The acceleration is rate of change of velocity or change in velocity per unit time interval.
- Velocity is a vector quantity hence a change in its magnitude or in direction or in both, will give the acceleration (or non uniform motion).
- **Average acceleration**
  - Let velocity of a particle at t_1 is v_1 and at t_2 it is $\vec{v}_2$. The change in velocity in time interval $(t_2 - t_1)$ is $(\vec{v}_2 - \vec{v}_1)$.

$$\therefore \quad a_{av} = \frac{v_2 - v_1}{t_2 - t_1}.$$

- For small time interval between t and $t + \Delta t$, change in velocity is taken $\Delta\vec{v}$ then average acceleration in the time interval Δt is, $a_{av} = \frac{\Delta v}{\Delta t}$.

- **Instantaneous acceleration**
  - If Δt approaches zero, the average acceleration becomes the instantaneous acceleration (or acceleration) $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

    i.e., instantaneous acceleration is the time derivative of velocity.

EQUATIONS OF MOTION

- If $\vec{u}$ = initial velocity of the body,

  $\vec{a}$ = uniform acceleration of the body,

  $\vec{s}$ = displacement in time t.

  $\vec{v}$ = velocity of the body after a time t, then the following equations hold good, in order to describe the motion of the body.

- If the motion is along a straight line,
  - $v = u + at$ ..... without s
  - $s = ut + \frac{1}{2}at^2$ ..... without v
  - $v^2 = u^2 + 2as$ ..... without t
  - $s = vt - \frac{1}{2}at^2$ ..... without u
  - $s = \frac{1}{2}(u + v)t$ ..... without a
  - Distance travelled in n^{th} sec, $s_n = u + \frac{a}{2}(2n - 1)$

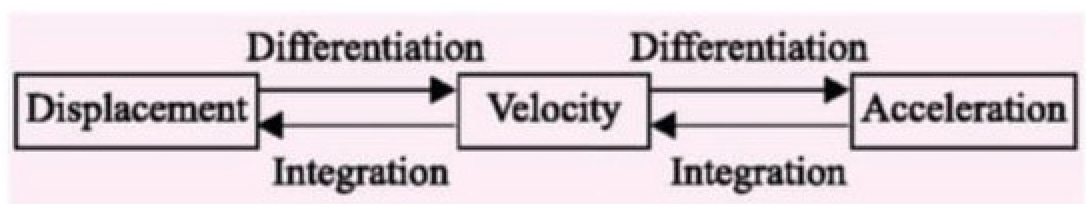
- If the motion is not along a straight line
  - $\vec{v} = \vec{u} + \vec{a}t$
  - $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
  - $\vec{v} \cdot \vec{v} = \vec{u} \cdot \vec{u} + 2\vec{a} \cdot \vec{s}$

- Equations of motion are valid only when acceleration remains constant during motion, otherwise we have to write the equations as under :

  $v = \int a(t)dt$ and $s = \int v(t)dt$

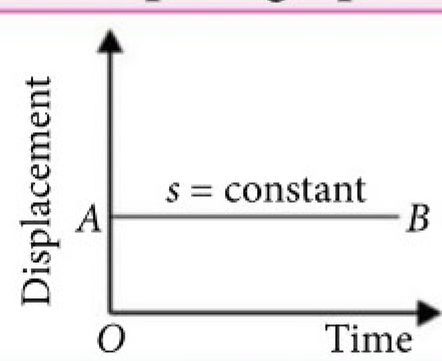
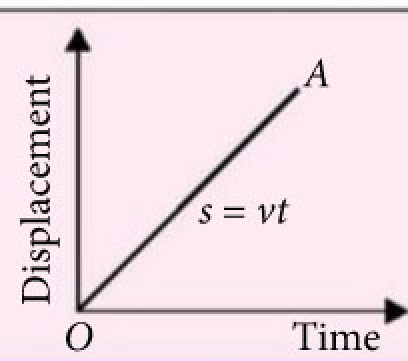
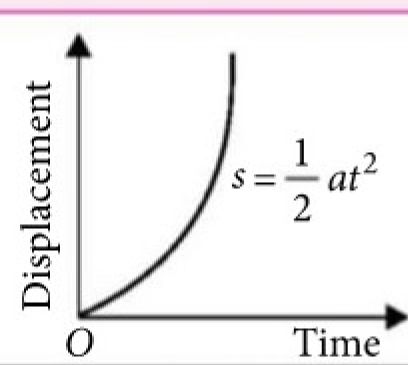
EFFECTIVE USE OF MATHEMATICAL TOOLS IN SOLVING PROBLEMS OF ONE-DIMENSIONAL MOTION

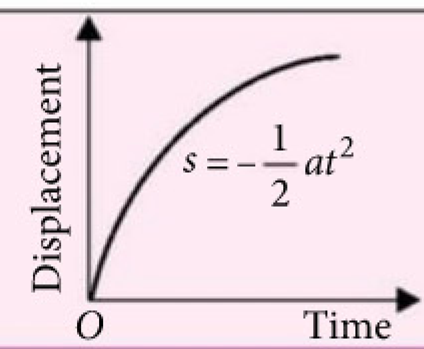
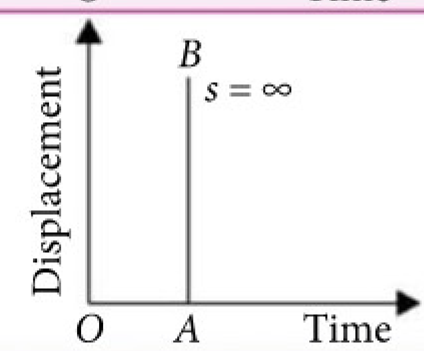
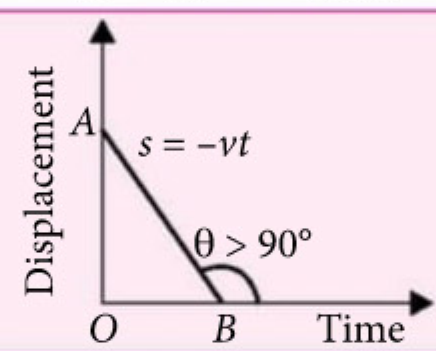
- If displacement-time equation is given, we can get velocity-time equation with the help of differentiation. Again, we can get acceleration-time equation with the help of differentiation.
- If acceleration-time equation is given, we can get velocity-time equation by integration. From velocity-time equation, we can get displacement-time equation by integration.



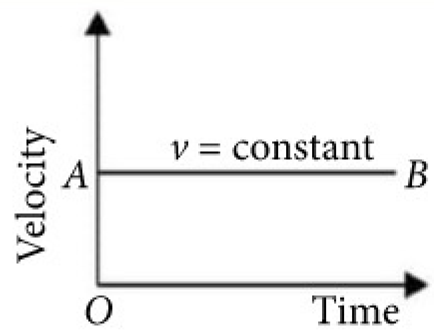
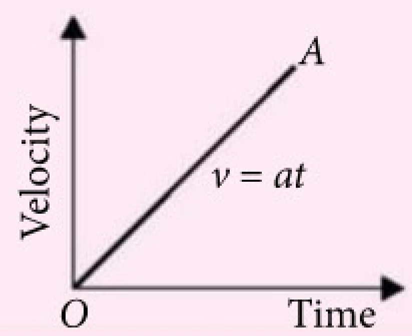
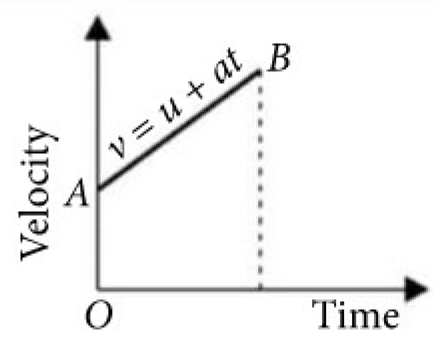
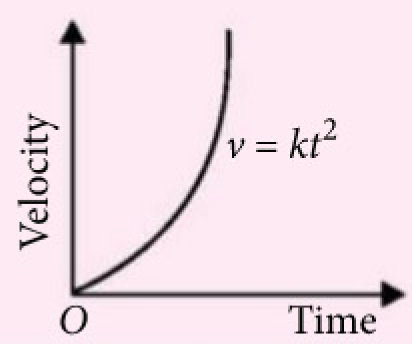
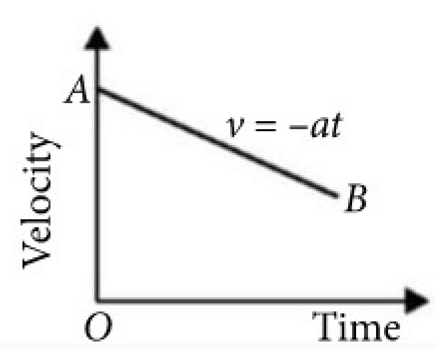
KINEMATIC GRAPHS

- **Displacement-Time Graph for Various Types of Motion of a Body**

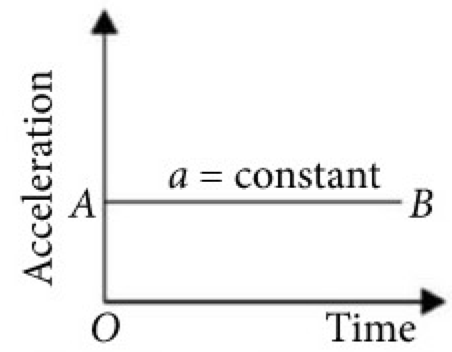
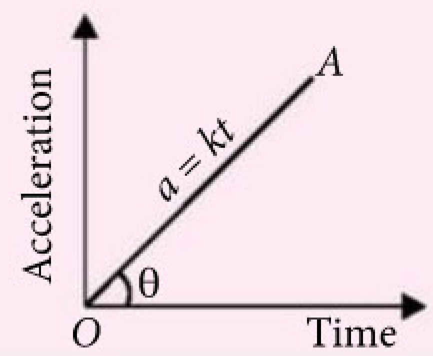
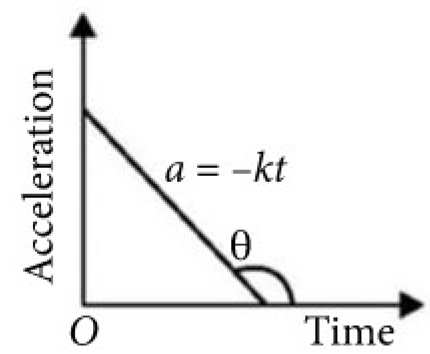
Description of motion	Shape of graph	Feature of graph
For a stationary body, the displacement-time graph is a straight line AB parallel to the time axis.		The slope of straight line AB (representing instantaneous velocity) is zero.
When a body is moving with constant velocity, the displacement-time graph will be a straight line OA, inclined to time axis.		Greater is the slope of straight line OA, greater will be the velocity.
When a body is moving with a constant acceleration, the displacement-time graph is a curve which bend upwards.		The slope of displacement-time curve (*i.e.*, instantaneous velocity) increases with time.

When a body is moving with constant retardation, the displacement-time graph is a curve which bend downwards.		The slope of displacement-time curve (*i.e.*, instantaneous velocity) decreases with time.
When a body is moving with infinite velocity, the displacement-time curve is a straight line *AB* parallel to displacement axis.		Such a motion of the body is never possible.
When a body returns back towards the original point of reference while moving with uniform negative velocity, the displacement-time graph is an oblique straight line *AB*, making an angle $\theta > 90°$ with the time axis.		The displacement of the body decreases with time with respect to the reference point, till it becomes zero.

- **Velocity-Time Graph for Various Types of Motion of a Body**

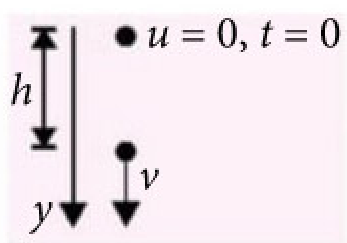
Description of motion	Shape of graph	Feature of graph
When a body is moving with a constant velocity, the velocity-time graph is a straight line *AB* parallel to time axis.		The slope of this graph (representing the instantaneous acceleration) is zero.
When a body is moving with a constant acceleration and its initial velocity is zero, the velocity-time graph is an oblique straight line, passing through the origin.		Greater is the slope of straight line *OA*, greater will be the instantaneous acceleration.
When a body is moving with a constant acceleration and its initial velocity is not zero, the velocity-time graph is an oblique straight line *AB* not passing through the origin.		(i) Here *OA* represents the initial velocity of the body. (ii) The area enclosed by the velocity-time graph with time axis represents the distance travelled by the body.
When a body is moving with increasing acceleration, the velocity-time graph is a curve which bend upwards.		The slope of velocity-time graph (*i.e.* instantaneous acceleration increases with time.
When a body is moving with a constant retardation and its initial velocity is not zero, the velocity-time graph is an oblique straight line not passing through the origin.		The slope of this straight line with time axis, makes an angle $\theta > 90°$.

- **Acceleration-Time Graph for Various Types of Motion of a Body**

Description of motion	Shape of graph	Feature of graph
When a body is moving with constant acceleration, the acceleration-time graph is a straight line *AB* parallel to time axis.		The area enclosed by acceleration-time graph for the given time gives the velocity of the body.
When a body is moving with constant increasing acceleration, the acceleration-time graph is a straight line *OA*.		The body is moving with constant acceleration and slope of straight line *OA*, makes an angle $\theta < 90°$ with time axis.
When a body is moving with constant decreasing acceleration, the acceleration-time graph is a straight line.		The body is moving with negative acceleration and slope of straight line makes an angle $\theta > 90°$ with time axis.

☞ VERTICAL MOTION UNDER GRAVITY

- If air resistance is neglected and a body is freely moving along vertical line near the earth's surface then an acceleration acts downward which is 9.8 m s^{-2} or 980 cm s^{-2}.
- **Freely falling body released from a height *h* above the ground**
  - ➢ Taking initial position asorigin and direction of motion (*i.e.*, downward direction) positive *y*-axis.



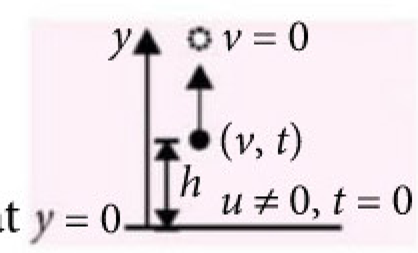
    As body is just released/dropped, $u = 0$
    acceleration along $+y$ axis, $a = g$
  - ➢ The body acquires velocity *v*(downward) after falling a distance *h* in time *t*, then equations of motion become

$$v = gt \qquad \left(\text{or} \quad t = \frac{v}{g}\right)$$

$$h = \frac{1}{2}gt^2 \qquad \left(\text{or} \quad t = \sqrt{\frac{2h}{g}}\right)$$

$$v^2 = 2gh \qquad \left(\text{or} \quad v = \sqrt{2gh} \quad \text{or} \quad h = v^2/2g\right)$$

- **Body projected vertically upward**
  - ➢ Take initial position asorigin and direction of motion (*i.e.*, vertically upward) as positive *y*-axis.



    $v = 0$ at maximum height, at
    $a = -g$ (because directed downward)
  - ➢ Equations of motion of the particle at any latter time *t* become

$$v = u - gt$$

$$h = ut - \frac{1}{2}gt^2$$

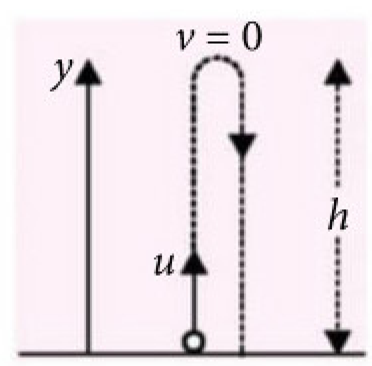
$$v^2 = u^2 - 2gh$$

  - ➢ At time $t = T$, $u = gT$

$$h_{max} = \frac{1}{2}gT^2 \text{ or, } T = \sqrt{\frac{2h_{max}}{g}}$$

$$u^2 = 2gh_{max} \qquad \text{or,} \quad h_{max} = \frac{u^2}{2g}$$

- After attaining maximum height, body comes back at the ground. During complete flight acceleration is constant, $a = -g$.
- Time taken during up flight and down flight are equal.



- Time for one side, $T = \frac{u}{g}$

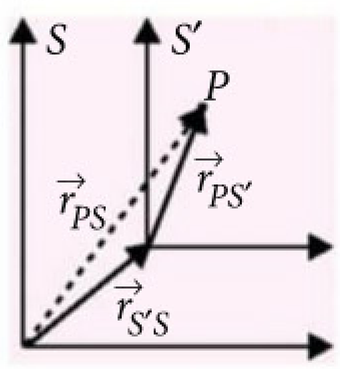
  and total flight time $= 2T = \frac{2u}{g}$

- At each equal height from ground speed of body will be same whether going up or coming down.

☞ RELATIVE MOTION

- There is no meaning of motion without reference or observer. If reference is not mentioned then we take the ground as a reference of motion. Velocity or displacement of the particle with respect to ground is called actual velocity or actual displacement of the body.
- If we describe the motion of a particle with respect to an object which is also moving w.r.t. ground then velocity of particle w.r.t. ground is its actual velocity ($\vec{v}_{act}$) and velocity of particle w.r.t. moving object is its relative velocity ($\vec{v}_{rel}$) and the velocity of moving object (w.r.t. ground) is the reference velocity ($\vec{v}_{ref}$).
- In the figure let S is ground frame and S' is frame of moving object. Position of particle P relative to frame S is $\vec{r}_{PS}$ while position of frame S' relative to frame S is $\vec{r}_{S'S}$ at a moment. According to vector law of addition $\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$.
- Differentiate the equation w.r.t. time,

$$\frac{d\vec{r}_{PS}}{dt} = \frac{d\vec{r}_{PS'}}{dt} + \frac{d\vec{r}_{S'S}}{dt} \left(\text{but } \vec{v} = \frac{d\vec{r}}{dt}\right)$$



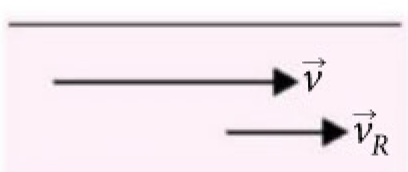
So, $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$

i.e., $\vec{v}_{\text{actual}} = \vec{v}_{\text{relative}} + \vec{v}_{\text{reference}}$

or $\vec{v}_{\text{relative}} = \vec{v}_{\text{actual}} - \vec{v}_{\text{reference}}$

- **Swimming in the river**
  - ➢ A man can swim with velocity $\vec{v}$ *i.e.*, the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$
  - ➢ If the swimming is in the direction of flow of water or along the downstream, then

$v_m = v + v_R$

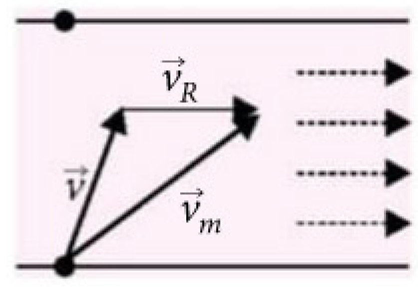


  - ➢ If the swimming is in the direction opposite to the flow of water or along the upstream, then

$v_m = v - v_R$

  - ➢ If the man swims across the river *i.e.*, $\vec{v}$ and $\vec{v}_R$ are not collinear then use the vector algebra

$\vec{v}_m = \vec{v} + \vec{v}_R$

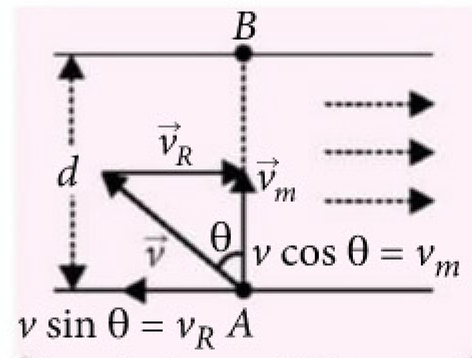


Case (I) : For shortest path, resultant velocity $\vec{v}_m = (\vec{v} + \vec{v}_R)$ is in the direction of displacement AB.

To reach at B, $v \sin\theta = v_R$

$\Rightarrow \sin\theta = \frac{v_R}{v}$

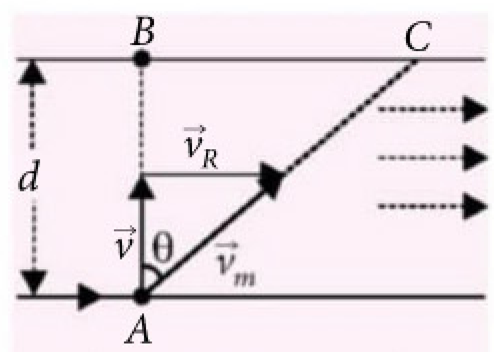
Component of velocity along $AB = v\cos\theta$



(For mimimum displacement)

So time taken, $T = \frac{d}{v\cos\theta} = \frac{d}{\sqrt{v^2 - v_R^2}}$

Case (II) : For minimum time, man should start swimming perpendicular to water current. Due to effect of river velocity, man will reach at point C along resultant velocity, *i.e.*, his displacement will not be minimum but time taken to cross the river will be minimum.



(For minimum time)

$$t_{\min} = \frac{d}{v}$$

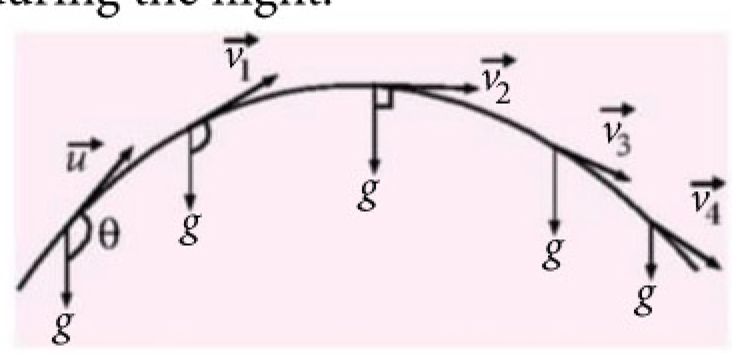
In time $t_{\min}$ swimmer travels distance BC along the river with speed of river v_R.

$\therefore\ BC = t_{\min} v_R$ = distance travelled along river flow

$= \text{drift of man} = \frac{d}{v} v_R$.

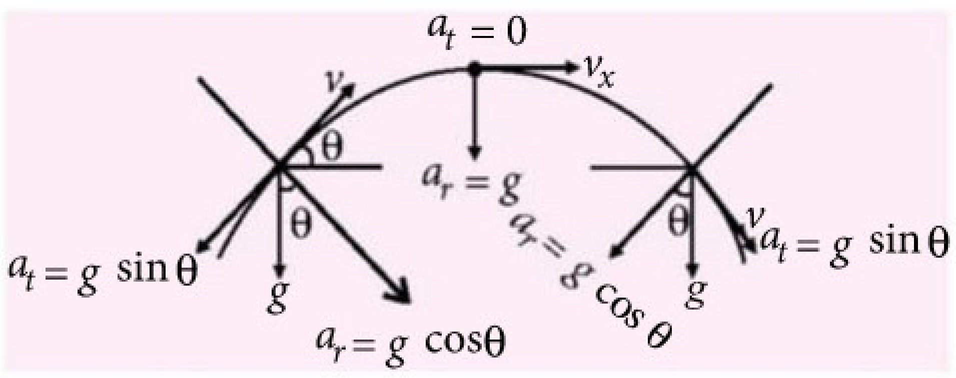
☞ PROJECTILE MOTION

- A particle thrown in the space which moves under the effect of gravity only is called a projectile. The motion of this projectile is referred as projectile motion.
- Projectile motion = Horizontal motion + Vertical motion
- Angle between velocity vector and acceleration vector during the flight.

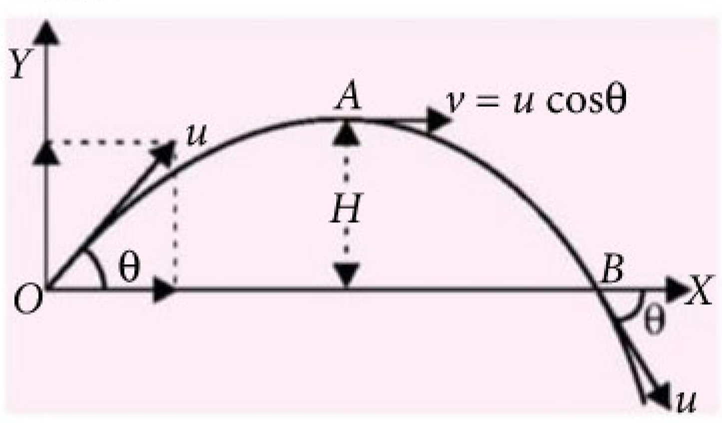


- In ascending motion θ is an obtuse angle. At the top of the flight θ = 90°, and during descending motion, θ is an acute angle. During the flight θ decreases continuously with increase in time and it always lies between 0° and 180°.

- In projectile motion, magnitude as well as direction of velocity continuously changes so there must be presence of tangential (a_t) as well as radial component (a_r) acceleration.



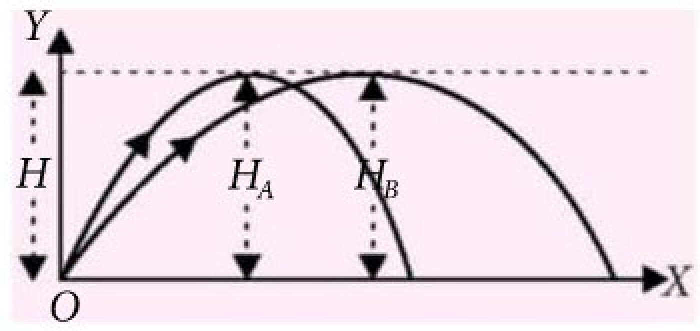
- In ascending motion, a_t is against v, so v decreases and in descending motion a_t is in the direction of v, so v increases.
- Since deceleration in y-component of velocity in ascending motion is equal to acceleration in y-component of velocity during descending motion, so time of ascending motion is equal to the time of descending motion and magnitude of y-component of velocity at same height in ascending as well as in descending motion is same but opposite in directions.
- At maximum height, $v_y = 0$ and $v_x = u_x = u\cos\theta$ so that at maximum height $v = \sqrt{v_x^2 + v_y^2} = u\cos\theta$
- When particle again returns to ground at point B, its y coordinate is zero and its magnitude of velocity is u at angle θ with ground. Angular change (or difference) between initial velocity and final velocity = 2θ.



Initial velocity $\vec{u}_i = u\cos\theta\hat{i} + u\sin\theta\hat{j}$
Final velocity, $\vec{u}_f = u\cos\theta\hat{i} - u\sin\theta\hat{j}$
Total change in its velocity $= |\Delta\vec{u}| = 2u\sin\theta$
Total change in momentum $= m|\Delta\vec{u}| = 2mu\sin\theta$

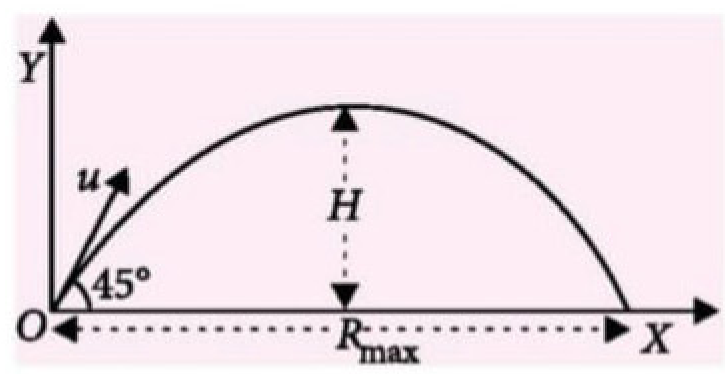
- If K_0 is initial kinetic energy, then kinetic energy at highest point = $K_0\cos^2\theta$

- $T = \dfrac{2u_y}{g}, H = \dfrac{u_y^2}{2g}, R = \dfrac{2u_x u_y}{g}$
  - T and H depend only upon initial vertical speed u_y.
  - If two projectiles thrown in different directions, have equal time of flight then their initial vertical speeds are same so that their maximum height attained is also same.
  - If $H_A = H_B$ then $(u_y)_A = (u_y)_B$ and $T_A = T_B$



- When two projectiles are thrown with equal speeds at angle θ and (90° – θ), then their ranges are equal but maximum heights attained are different and time of flights are also different.
- For maximum range, θ = 45°

and $R_{max} = \dfrac{u^2 \sin 2(45°)}{g} = \dfrac{u^2 \sin 90°}{g} \Rightarrow R_{max} = \dfrac{u^2}{g}$



Here $H = \dfrac{u^2 \sin^2 45°}{2g} = \dfrac{u^2}{4g}$

$\therefore \quad \dfrac{R_{max}}{H} = 4 \Rightarrow R_{max} : H = 4:1$

- For $R = H$

$\Rightarrow \dfrac{u^2(2\sin\theta\cos\theta)}{g} = \dfrac{u^2\sin^2\theta}{2g} \Rightarrow 2\cos\theta = \dfrac{\sin\theta}{2}$

$\Rightarrow \tan\theta = 4 \quad \therefore \quad \theta = \tan^{-1}(4) = 76°$

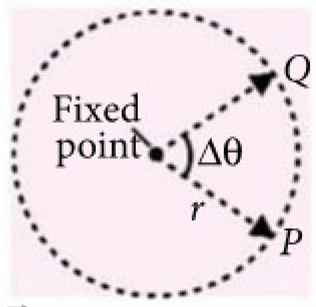
CIRCULAR MOTION

- When a particle moves in a plane such that its distance, from fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed (or moving) point. That point is called centre and the distance is called radius of circular path.
- **Angular Displacement :** Angle traced by position vector of a particle moving w.r.t. some fixed point is

called angular displacement.

$\Delta\theta$ = angular displacement

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}; \Delta\theta = \frac{ArcPQ}{r}$$



- Small angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement is scalar quantity.
- Its direction is perpendicular to plane of rotation and given by right hand screw rule.
- It is dimensionless and its S.I. unit is radian while other units are degree or revolution.
  2π radian = 360° = 1 revolution

- **Angular Velocity :** It is defined as the rate of change of angular displacement of moving particle.

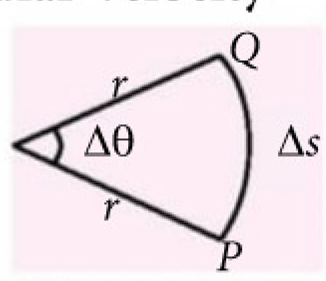
$$\omega = \frac{\text{Angle traced}}{\text{Time taken}} = \lim_{\Delta t \to \infty} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Its direction is same as that of angular displacement *i.e.*, perpendicular to the plane of rotation and along the axis according to right hand screw rule.
- Its unit is radian/second.

- **Relation between Linear and Angular Velocity**

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$\Delta\theta = \frac{\Delta s}{r} \quad \text{or} \quad \Delta s = r\Delta\theta$$



$$\therefore \quad \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt} \text{ or } v = \omega r$$

In vector form

$\vec{v} = \vec{\omega} \times \vec{r}$

(direction of $\vec{v}$ is found according to right hand rule)

- **Average Angular Velocity (ω_{av})**

$$\omega_{av} = \frac{\text{Total angle of rotation}}{\text{Total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

$$= \frac{2\pi}{T} = 2\pi n$$

where θ_1 and θ_2 are angular positions of the particle at instants t_1 and t_2 respectively.

- **Instantaneous Angular Velocity**

The angular velocity at some particular instant of time is called instantaneous angular velocity.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}; \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

- **Angular Acceleration**

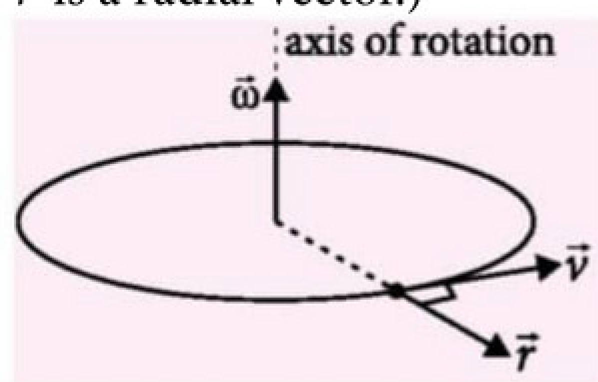
Rate of change of angular velocity is called angular acceleration.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad ; \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

- It is an axial vector quantity. Its direction is along the axis according to right hand screw rule.

- **Relation between Angular and Linear Acceleration**

$\vec{v} = \vec{\omega} \times \vec{r}$ ($\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector.)



but $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

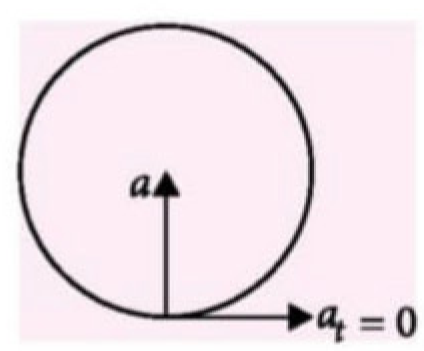
or $\quad \vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

or $\quad \vec{a} = \vec{a}_T + \vec{a}_C$

- ($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration and $\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)
- $\vec{a}_T$ and $\vec{a}_C$ are two mutually perpendicular components of net linear acceleration.

UNIFORM CIRCULAR MOTION

- When a body moves along a circular path with uniform speed, its motion is said to be uniform circular motion.
  - Position vector ($\vec{r}$) is always perpendicular to the velocity vector ($\vec{v}$) *i.e.* $\vec{r} \cdot \vec{v} = 0$
  - Velocity vector is always perpendicular to the acceleration. $\vec{v} \cdot \vec{a} = 0$
- For circular motion, force towards centre (centripetal force) must act so that direction of $\vec{v}$ keeps on changing.

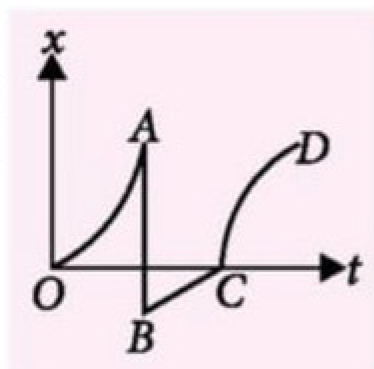


- The work done by centripetal force is always zero.
- Kinetic energy = constant
- Since $|\vec{v}|$ = constant, so tangential acceleration $a_t = 0$
- In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

WRAP it up!

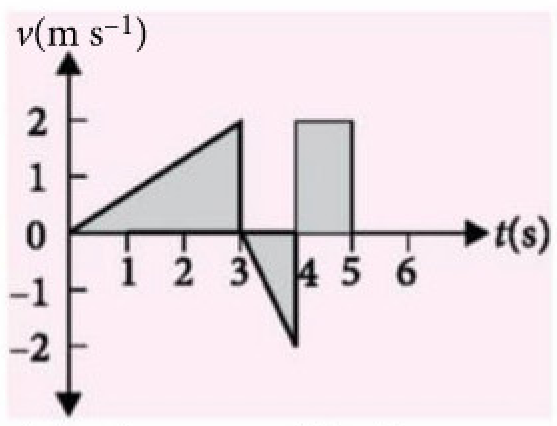
1. A ball falls from 20 m height on the floor and rebounds to 5 m. Time of contact is 0.02 s. Find the acceleration during impact.
(a) 1200 m s^{-2} (b) 1000 m s^{-2}
(c) 2000 m s^{-2} (d) 1500 m s^{-2}

2. The displacement-time (x-t) graph of a body is shown in figure. The body is accelerated along the path



(a) *OA* only
(b) *BC* only
(c) *CD* only
(d) *OA* and *CD* only

3. The velocity versus time graph of a body moving in a straight line is shown in figure. The displacement of the body in 5 s is



(a) 3 m (b) 5 m (c) 4 m (d) 2 m

4. An aeroplane is flying in a horizontal direction with a velocity of 360 km h^{-1} and at a height of 1960 m. When it is vertically above the point *A* on the ground, a body is dropped from it. The body strikes the ground at a point *B*. The distance *AB* is
(a) $2000\sqrt{2}$ m (b) 2000 m
(c) $1000\sqrt{2}$ m (d) 1000 m

5. A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 s is
(a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$
(c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(n+1)}{n}$

6. Two cities C_1 and C_2 are connected on the opposite ends of a long straight parallel track. The cities are connected by a train service as well as a bus service. The trains leave with constant speed v for either city at regular frequency of one train every x minute. The buses ply on a parallel road at a constant speed of 30 km h^{-1}. A bus passenger going from city C_1 to city C_2 observes a train going past him every 20 minutes while a train goes in the opposite direction every 10 minutes. What are the values of x and v ?
(a) x = 15 min, v = 90 km h^{-1}
(b) x = 13 min 20 s, v = 90 km h^{-1}
(c) x = 15 min, v = 75 km h^{-1}
(d) x = 13 min 20 s, v = 70 km h^{-1}

7. A particle is projected with a certain velocity at two different angles of projection with respect to the horizontal plane so as to have the same range R on the horizontal plane. If t_1 and t_2 are the times taken for the two paths, then which one of the following relations is correct?
(a) $t_1t_2 = 2R/g$ (b) $t_1t_2 = R/g$
(c) $t_1t_2 = R/2g$ (d) $t_1t_2 = 4R/g$

8. The position of a particle moving in the x-y plane at any time t is given by; $x = (3t^2 - 6t)$ metres; $y = (t^2 - 2t)$ metres. Select the correct statement.
(a) Acceleration is zero at $t = 0$
(b) Velocity is zero at $t = 0$
(c) Velocity is zero at $t = 1$ s
(d) Velocity and acceleration of the particle are never zero.

9. A stone tied to the end of a 1 m long string, is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of acceleration of the stone?
(a) $\frac{\pi^2}{4}$ m s^{-2} and direction along the radius towards the centre.
(b) π^2 m s^{-2} and direction along the radius away from the centre.
(c) π^2 m s^{-2} and direction along the radius towards the centre.
(d) π^2 m s^{-2} and direction along the tangent to the circle.

10. Water drops fall at regular intervals from a tap which is 5.0 m above the ground. The third drop is leaving the tap at the instant the first drop reaches the ground. How far above the ground is the second drop at that instant?
(a) 1.25 m (b) 2.50 m (c) 3.75 m (d) 5.00 m

11. Two cars having masses m_1 and m_2 move in circles of radii r_1 and r_2 respectively. If they complete the circle in equal time, the ratio of their angular speed ω_1/ω_2 is

(a) $\frac{m_1}{m_2}$ (b) $\frac{r_1}{r_2}$ (c) $\frac{m_1 r_1}{m_2 r_2}$ (d) 1

12. A man running on a horizontal road at 8 km h^{-1} finds the rain falling vertically. He increases his speed to 12 km h^{-1} and finds that the drops make angle 30° with the vertical. What is the speed of the rain ?

(a) 8 km h^{-1} (b) $4\sqrt{7}$ km h^{-1}

(c) $8\sqrt{3}$ km h^{-1} (d) $\sqrt{7}$ km h^{-1}

13. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$, where ω is a constant. Which of the following is true?

(a) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.

(b) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin.

(c) Velocity and acceleration both are perpendicular to $\vec{r}$.

(d) Velocity and acceleration both are parallel to $\vec{r}$.

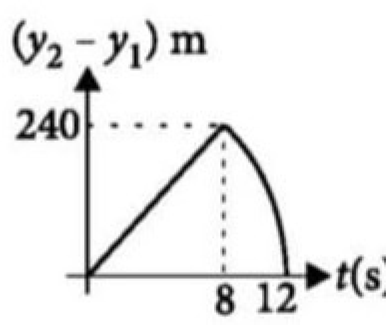
14. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration (in m s^{-2}) at the end of 2.0 s is approximately

(a) 6.0 (b) 3.0 (c) 8.0 (d) 7.0

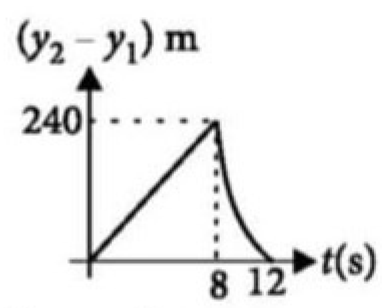
15. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m s^{-1} and 40 m s^{-1} respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first stone?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10$ m s^{-2})

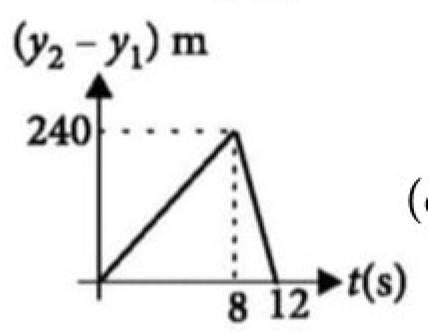
(a)



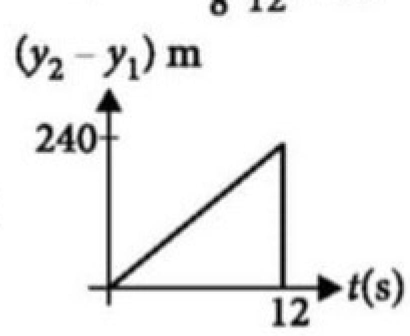
(b)



(c)

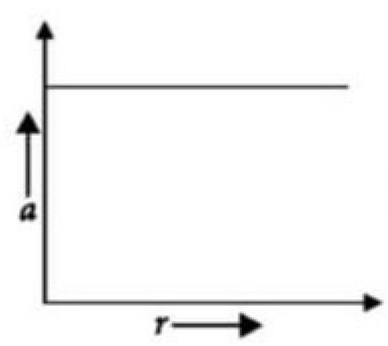


(d)

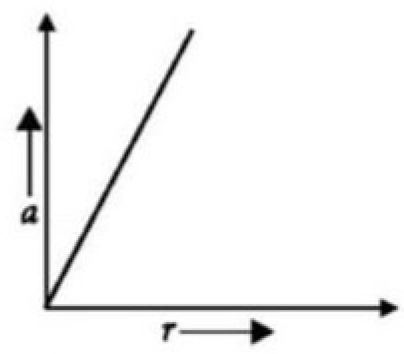


16. If a body is moving in a circular path maintains constant speed of 10 m s^{-1}, then which of the following correctly describes relation between acceleration and radius ?

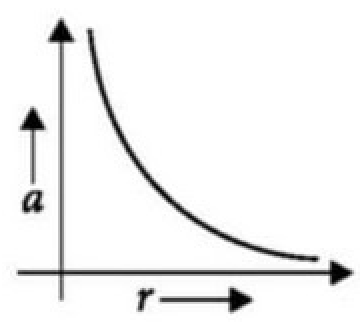
(a)



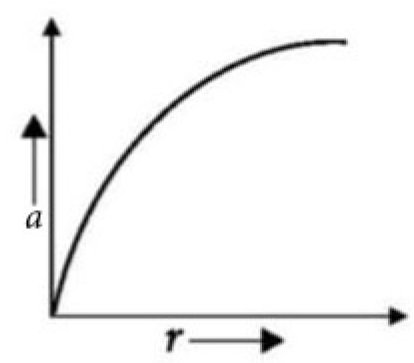
(b)



(c)



(d)



17. The position vector of a particle $\vec{R}$ as a function of time is given by $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$ Where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x and y-directions, respectively. Which one of the following statements is wrong for the motion of the particle?

(a) Magnitude of the velocity of particle is 8 m s^{-1}.

(b) Path of the particle is a circle of radius 4 m.

(c) Acceleration vector is along $-\vec{R}$.

(d) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle.

18. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is

(a) $gH = (n - 2)u^2$ (b) $2gH = n^2u^2$

(c) $gH = (n - 2)^2u^2$ (d) $2gH = nu^2(n - 2)$

19. A projectile is fired from the surface of the earth with a velocity of 5 m s^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 m s^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet (in m s^{-2}) is (Given $g = 9.8$ m s^{-2})

(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8

20. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m s^{-1} where, $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10$ m s^{-2}, the equation of its trajectory is

(a) $4y = 2x - 25x^2$ (b) $y = x - 5x^2$
(c) $y = 2x - 5x^2$ (d) $4y = 2x - 5x^2$

SOLUTIONS

1. **(d) :** The acceleration during impact,

$$a = \frac{v_2 - (-v_1)}{t} = \frac{v_2 + v_1}{t} = \frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{t}$$

$$= \frac{\sqrt{2\times10\times20} + \sqrt{2\times10\times5}}{0.02} = \frac{20+10}{0.02} = 1500 \text{ m s}^{-2}$$

2. **(a) :** The slope of x-t graph gives velocity and the slope of the graph is increasing in part OA only.

3. **(c) :** Displacement = area of bigger triangle – area of smaller triangle + area of rectangle

$$= \left[\frac{1}{2}(3\times2) - \frac{1}{2}(1\times2) + (1\times2)\right] = 4 \text{ m}$$

4. **(b) :** Time taken by the body to strike the ground is given by $h = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times1960}{9.8}} = 20 \text{ s}$

Velocity of the aeroplane in horizontal direction,

$v_{x0} = 360 \text{ km h}^{-1} = 100 \text{ m s}^{-1}$

Distance $AB = v_{x0} \times t = (100 \times 20) \text{ m} = 2000 \text{ m}$

5. **(a) :** Since, $a = \frac{v-u}{t} = \frac{v-0}{n} = \frac{v}{n}$

Displacement in last 2 s,

$$s_n - s_{n-2} = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= 2a(n-1) = 2\times\frac{v}{n}(n-1) = \frac{2v(n-1)}{n}$$

6. **(b) :** If v (in km h^{-1}) is the constant speed of the trains then the distance between the successive trains

$$= v\times\frac{x}{60} = \frac{vx}{60} \text{ km}$$

When a train moves in the same direction as that of the bus passenger, $\frac{vx/60}{(v-30)} = \frac{20}{60}$

or $vx = 20(v - 30)$ ...(i)

When a train moves in a direction opposite to the bus passenger,

$\frac{vx/60}{(v+30)} = \frac{10}{60}$ or $vx = 10(v + 30)$ ...(ii)

From eqns. (i) and (ii),

$20(v - 30) = 10(v + 30)$ or $v = 90 \text{ km h}^{-1}$

From eqn. (i),

$90x = 20(90 - 30) = 1200$ or $x = 13$ min 20 s

7. **(a) :** R is same for angles of projection θ and $(90° - \theta)$, *i.e.*, $R = v_0^2 \sin 2\theta / g$

As $t_1 = \frac{2v_0 \sin\theta}{g}$ and $t_2 = \frac{2v_0 \sin(90° - \theta)}{g} = \frac{2v_0\cos\theta}{g}$,

$$t_1t_2 = \frac{4v_0^2 \sin\theta\cos\theta}{g^2} = \frac{2}{g}\left(\frac{v_0^2 \sin 2\theta}{g}\right) = \frac{2R}{g}$$

8. **(c) :** Since, $x = 3t^2 - 6t$ $\therefore$ $v_x = \frac{dx}{dt} = 6t - 6$

At $t = 1$ s, $v_x = 6 \times 1 - 6 = 0$

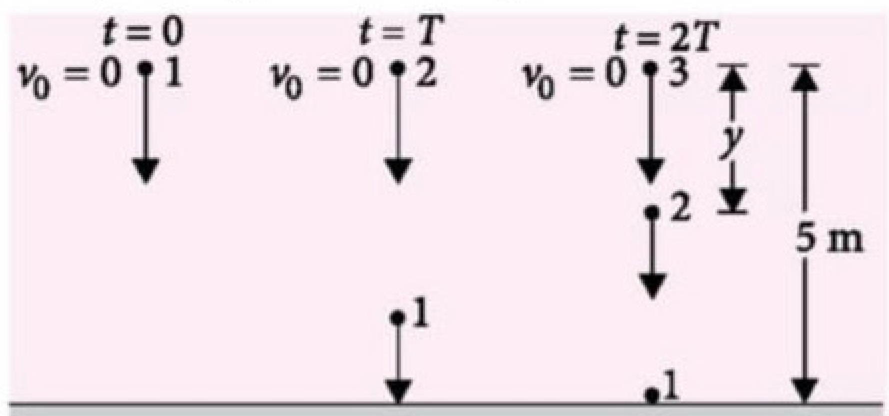
Also, $y = t^2 - 2t$ $\therefore$ $v_y = \frac{dy}{dt} = 2t - 2$

At $t = 1$ s, $v_y = 2 \times 1 - 2 = 0$

9. **(c) :** $a = r\omega^2 = r(2\pi n/t)^2$

$$= r\times\frac{4\pi^2n^2}{t^2} = \frac{1\times4\times\pi^2\times(22)^2}{(44)^2} = \pi^2 \text{ m s}^{-2}$$

10. **(c) :** Let T be the time interval between the drops (1, 2, 3) falling from the tap as shown in the figure.



Since distance covered by the first drop in time $2T$ is 5 m,

$$5 = \frac{1}{2}g(2T)^2 = 2gT^2 \quad \text{...(i)}$$

Further, distance covered by the second drop in time T (from $t = T$ to $t = 2T$),

$$y = \frac{1}{2}gT^2 \quad \text{...(ii)}$$

From eqns. (i) and (ii), $y = 1.25$ m

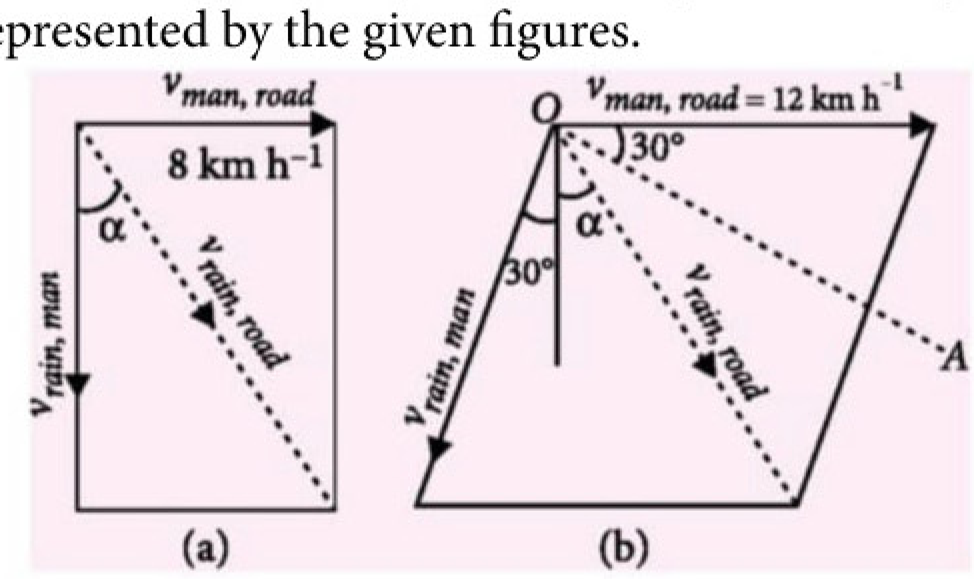
Distance of the second drop from the ground

$= 5 - y = 5 - 1.25 = 3.75$ m

11. **(d)**

12. **(b):** We have, $\vec{v}_{rain,\,road} = \vec{v}_{rain,\,man} + \vec{v}_{man,\,road}$ ...(i)

The two situations given in the problem may be represented by the given figures.



$v_{rain,\ road}$ is same in magnitude and direction in both the figures.
Taking horizontal components in equation (i) for figure (a)

$$v_{rain, road} \sin\alpha = 8 \quad \text{...(ii)}$$

Now consider figure (b). Draw a line $OA \perp v_{rain,\ man}$ as shown.
Taking components in equation (i) along the line OA.
$v_{rain,\ road} \sin(30° + \alpha) = 12 \cos 30°$ ...(iii)
From (ii) and (iii),

$$\frac{\sin(30°+\alpha)}{\sin\alpha} = \frac{12\times\sqrt{3}}{8\times 2} \quad \text{or} \quad \cot\alpha = \frac{\sqrt{3}}{2}$$

From (ii), $v_{rain, road} = \dfrac{8}{\sin\alpha} = 4\sqrt{7}$ km h^{-1}

13. (a) : Given, $\vec{r} = \cos\omega t\ \hat{x} + \sin\omega t\ \hat{y}$

$$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = -\omega\sin\omega t\ \hat{x} + \omega\cos\omega t\ \hat{y}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2\cos\omega t\ \hat{x} - \omega^2\sin\omega t\ \hat{y} = -\omega^2\vec{r}$$

Since position vector $(\vec{r})$ is directed away from the origin, so, acceleration $(-\omega^2\vec{r})$ is directed towards the origin. Also,
$\vec{r}\cdot\vec{v} = -\omega \sin\omega t \cos\omega t + \omega \sin\omega t \cos\omega t = 0$
$\Rightarrow \quad \vec{r} \perp \vec{v}$

14. (c) : Given, $r = 50$ cm $= 0.5$ m, $\alpha = 2.0$ rad s^{-2}, $\omega_0 = 0$
At the end of 2 s,
Tangential acceleration, $a_t = r\alpha = 0.5 \times 2 = 1$ m s^{-2}
Radial acceleration, $a_r = \omega^2 r = (\omega_0 + \alpha t)^2 r$
$= (0 + 2 \times 2)^2 \times 0.5 = 8$ m s^{-2}
$\therefore$ Net acceleration,

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8 \text{ m s}^{-2}$$

15. (a) : Using $h = ut + \frac{1}{2}at^2$

For first stone, $y_1 = 10t - \frac{1}{2}gt^2$

For second stone, $y_2 = 40t - \frac{1}{2}gt^2$

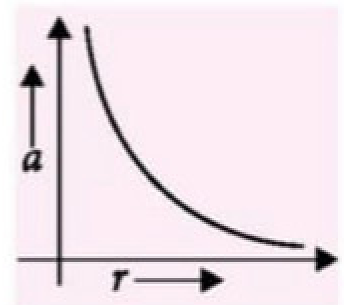
Relative position of the second stone with respect to the first stone $\Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2$
$= 30t$
Therefore, it will be a straight line.
After 8 seconds, first stone reaches to the ground, *i.e.*, $y_1 = -240$ m

$$\therefore \quad \Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 + 240$$

Therefore, it will be a parabolic curve till the second stone reaches to the ground.

16. (c) : Speed $v = 10$ m s^{-1}
We know, centripetal acceleration is given by,

$$a = \frac{v^2}{r}$$



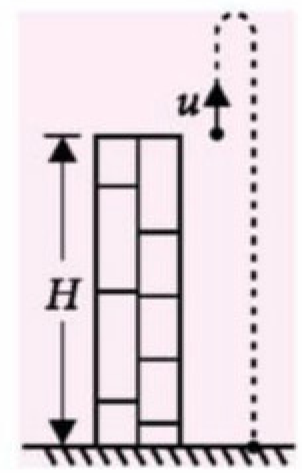
$\because \quad |\vec{v}| = \text{constant}$

so, $a \propto \frac{1}{r}$ or ar = constant
This represents a rectangular hyperbola.

17. (a)

18. (d): Time taken by the particle to reach the top most point is,

$$t = \frac{u}{g} \quad \text{... (i)}$$



Time taken by the particle to reach the ground $= nt$
Using, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad -H = u(nt) - \frac{1}{2}g(nt)^2$$

$$\Rightarrow \quad -H = u\times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2 \quad \text{[using (i)]}$$

$$\Rightarrow \quad -2gH = 2nu^2 - n^2u^2 \Rightarrow 2gH = nu^2(n-2)$$

19. (a) : The equation of trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

where θ is the angle of projection and u is the velocity with which projectile is projected.
For equal trajectories and for same angles of projection, $\dfrac{g}{u^2}$ = constant

As per question, $\dfrac{9.8}{5^2} = \dfrac{g'}{3^2}$

where g' is acceleration due to gravity on the planet.

$$g' = \frac{9.8\times 9}{25} = 3.5 \text{ m s}^{-2}$$

20. (c) : Given : $u = \hat{i} + 2\hat{j}$
As $\vec{u} = u_x\hat{i} + u_y\hat{j} \quad \therefore \quad u_x = 1$ and $u_y = 2$
Also $x = u_x t$

$$\therefore \quad x = t \text{ and } y = u_y t - \frac{1}{2}gt^2$$

$$\therefore \ y = 2t - \frac{1}{2}\times 10\times t^2 = 2t - 5t^2$$

Equation of trajectory is $y = 2x - 5x^2$.

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 120

Kinematics

Time Taken : 60 Min.

NEET

Only One Option Correct Type

1. A parachutist after bailing out, falls 50 m without friction. When parachute opens, it decelerates at 2 m s^{-2}. He reaches the ground with a speed of 3 m s^{-1}. At what height did he bail out?
(a) 111 m (b) 293 m
(c) 182 m (d) 91 m

2. A person travelling eastward with a speed of 3 km h^{-1} finds that wind seems to blow from north. On doubling his speed, the wind appears to flow from north-east. Find the magnitude of the actual velocity of the wind.
(a) 4 km h^{-1} (b) $2\sqrt{3}$ km h^{-1}
(c) 3 km h^{-1} (d) $3\sqrt{2}$ km h^{-1}

3. What is the resultant of three coplanar forces 300 N at 0°, 400 N at 30° and 400 N at 150°?
(a) 400 N, 45° (b) 500 N, 50°
(c) 500 N, 53° (d) 550 N, 60°

4. A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is
(a) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ (b) $\frac{v}{2}\sqrt{1+\cos^2\theta}$
(c) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ (d) $v\cos\theta$

5. A body dropped from top of a tower falls through 40 m during the last two seconds of its fall. The height of the tower (in m) is [Take g = 10 m s^{-2}]
(a) 60 m (b) 45 m
(c) 80 m (d) 50 m

6. A stone with weight W is thrown vertically upwards into air from ground level with initial velocity v_0. If a constant force f due to air drag acts on the stone throughout flight, the speed of the stone just before impact with the ground is
(a) $v_0\left(\frac{W-f}{W+f}\right)^{1/2}$ (b) $v_0\left(\frac{W+f}{W-f}\right)^{1/2}$
(c) $v_0\left(\frac{W-f}{W}\right)^{1/2}$ (d) $v_0\left(\frac{W+f}{f}\right)^{1/2}$

7. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flights in the two cases, then the product of the two time of flights is directly proportional to
(a) $1/R^2$ (b) $1/R$
(c) R (d) R^2

8. At a height 0.4 m from the ground, the velocity of a projectile in vector form is $\vec{v} = (6\hat{i} + 2\hat{j})$ m s^{-1}. The angle of projection is
(a) 45° (b) 60°
(c) 30° (d) $\tan^{-1}(3/4)$

9. A threaded rod with 12 turns per cm and diameter 1.18 cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at the rate of 216 rpm. How long will it take for the bar to move 1.50 cm along the rod?
(a) 3 s (b) 5 s
(c) 7 s (d) 4 s

10. A cannon on a level plain is aimed at an angle θ above the horizontal and a shell is fired with a muzzle velocity v_0 towards a cliff at a distance R away. The height from the bottom at which the shell strikes the side walls of the cliff is

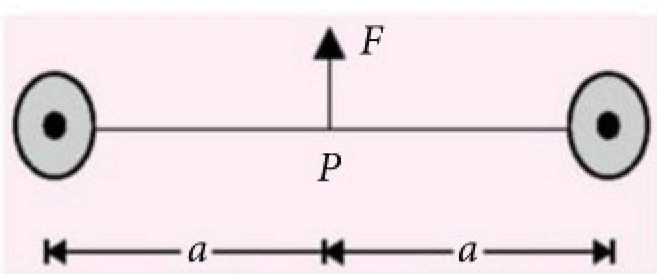
(a) $R\sin\theta - \dfrac{gR^2}{2v_0^2\sin^2\theta}$

(b) $R\cos\theta - \dfrac{gR^2}{2v_0^2\cos^2\theta}$

(c) $R\tan\theta - \dfrac{gR^2}{2v_0^2\cos^2\theta}$

(d) $R\tan\theta - \dfrac{gR^2}{2v_0^2\sin^2\theta}$

11. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance a from the centre P as shown in figure. Now the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other at the surfaces. The magnitude of acceleration, when the separation between them becomes $2x$ is



(a) $\dfrac{Fa}{2m\sqrt{a^2-x^2}}$ (b) $\dfrac{Fx}{2m\sqrt{a^2-x^2}}$

(c) $\dfrac{Fx}{\sqrt{2ma}}$ (d) $\dfrac{F\sqrt{a^2-x^2}}{2mx}$

12. Two trains take 3 s to pass one another when going in the opposite direction but only 2.5 s if the speed of one is increased by 50%. The time one would take to pass the other when going in the same direction at their original speeds is

(a) 10 s (b) 12 s

(c) 15 s (d) 18 s

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false.

(d) If both assertion and reason are false.

13. Assertion : The time of flight of a body becomes n times the original value if its speed is made n times.

Reason : This is due to the range of the projectile which becomes n times.

14. Assertion : When the displacement of a body is directly proportional to the square of the time. Then the body is moving with uniform acceleration.

Reason : The slope of velocity-time graph with time axis gives acceleration.

15. Assertion : If dot product and cross product of $\vec{P}$ and $\vec{Q}$ are zero, it implies that one of the vector $\vec{P}$ and $\vec{Q}$ must be null vector.

Reason : A null vector is a vector of zero magnitude.

JEE MAIN / ADVANCED

Only One Option Correct Type

16. A ball rolls from the edge of the top step of a stair case with horizontal speed 5 m s^{-1}. Each step is of 2 m high and 1m width. The ball hits

(a) 10^{th} step (b) 5^{th} step

(c) 2^{nd} step (d) 3^{rd} step

17. A policeman moving on a highway with a speed of 30 km h^{-1} fires a bullet at thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car?

(a) 120 m s^{-1} (b) 90 m s^{-1}

(c) 125 m s^{-1} (d) 105 m s^{-1}

18. The speed of a projectile when it is at its greatest height is $\sqrt{2/5}$ times its speed at half the maximum height. The angle of projection is

(a) 30° (b) 60°

(c) 45° (d) $\tan^{-1}(3/4)$

19. A particle is projected horizontally with a speed v_0 from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

(a) $\frac{2v_0^2 g \sec\theta}{\tan\theta}$ (b) $\frac{2v_0^2 g \tan\theta}{\sec\theta}$

(c) $\frac{2v_0^2 \tan\theta \sec\theta}{g}$ (d) $\frac{2v_0^2 g}{\tan\theta}$

More than One Options Correct Type

20. A train is passing through a platform of length 50 m with uniform velocity. It takes 15 s to cross the platform and 5 s to cross a man standing on the platform. Mark the correct option(s).

(a) The length of train is 25 m

(b) The length of train is 50 m

(c) The speed of train is 10 m s^{-1}

(d) The speed of train is 5 m s^{-1}

21. If a particle is projected with speed 20 m s^{-1} making angle 37° with horizontal. At $t = \frac{10}{3}$ s, (Take $\cos\theta = 4/5$)

(a) the velocity of the particle is horizontally directed

(b) the velocity is perpendicular to initial velocity

(c) the magnitude of velocity is $\frac{80}{3}$ m s^{-1}

(d) the magnitude of velocity is 16 m s^{-1}

22. Two cities A and B are connected by a regular bus service with buses plying in either direction every T seconds. The speed of each bus is uniform and equal to v_b. A cyclist cycles from A to B with a uniform speed of v_c. A bus goes past the cyclist in T_1 seconds in the direction A to B and every T_2 seconds in the direction B to A. Then

(a) $T_1 = \frac{v_b T}{v_b + v_c}$ (b) $T_2 = \frac{v_b T}{v_b - v_c}$

(c) $T_1 = \frac{v_b T}{v_b - v_c}$ (d) $T_2 = \frac{v_b T}{v_b + v_c}$

23. The coordinates of a particle moving in a plane given by $x = a\cos pt$ and $y = b\sin pt$ where $a, b\ (< a)$ and p are positive constants of appropriate dimensions. Then,

(a) the path of the particle is an ellipse
(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$
(c) the acceleration of the particle is always directed towards a fixed point
(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a

Numerical Value Type

24. A 200 m long train starts from rest at $t = 0$ with constant acceleration 4 cm s^{-2}. The head light of its engine is switched ON at $t = 60$ s and its tail light is switched ON at $t = 120$ s. The distance between these two events for an observer standing on platform is $(2)^n$ m. Find the value of n.

25. A stone is dropped from certain height which can reach the ground in 5 s. If the stone is stopped after 3 s of its fall and then allowed to fall again. Find the time taken (in second) by the stone to reach the ground for the remaining distance.

26. A particle is moving in a circle of radius R with constant speed. The time period (T) is 1 s. In a time $t = T/6$, if the difference between average speed and average velocity of the particle is 2 m s^{-1}, find the radius (in m) of the circle.

Comprehension Type

To a stationary man, rain appears to be falling at his back at an angle 30° with the vertical. As he starts moving forward with a speed of 0.5 m s^{-1}, he finds that the rain is falling vertically.

27. The speed of rain with respect to the stationary man is
(a) 0.5 m s^{-1} (b) 1.0 m s^{-1}
(c) $0.5\sqrt{3}$ m s^{-1} (d) 0.43 m s^{-1}

28. The speed of rain with respect to the moving man is
(a) 0.5 m s^{-1} (b) 1.0 m s^{-1}
(c) $0.5\sqrt{3}$ m s^{-1} (d) 0.45 m s^{-1}

Matrix Match Type

29. A ball is projected from the ground with velocity v such that its range is maximum.

Column-I	Column-II
(A) Velocity at half of the maximum height	(P) $\frac{\sqrt{3}v}{2}$
(B) Velocity at the maximum height	(Q) $\frac{v}{\sqrt{2}}$
(C) Change in its velocity when it returns to the ground	(R) $v\sqrt{2}$
(D) Average velocity when it reaches the maximum height	(S) $\frac{v}{2}\sqrt{\frac{5}{2}}$

	A	B	C	D
(a)	P	S	Q	R
(b)	P	Q	R	S
(c)	P	S	R	Q
(d)	R	P	S	Q

30. A balloon rises up with constant net acceleration of 10 m s^{-2} from the ground. After 2 s a particle drops from the balloon. After further 2 s match the following.
(Take $g = 10$ m s^{-2}).

Column-I	Column-II
(A) Height of particle from ground	(P) Zero
(B) Speed of particle	(Q) 10 SI units
(C) Height of the balloon	(R) 40 SI units
(D) Acceleration of particle	(S) 80 SI units

	A	B	C	D
(a)	R	P	S	Q
(b)	S	P	R	Q
(c)	P	S	Q	R
(d)	P	Q	R	S

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