N-body Problem: Analytical and Numerical Approaches

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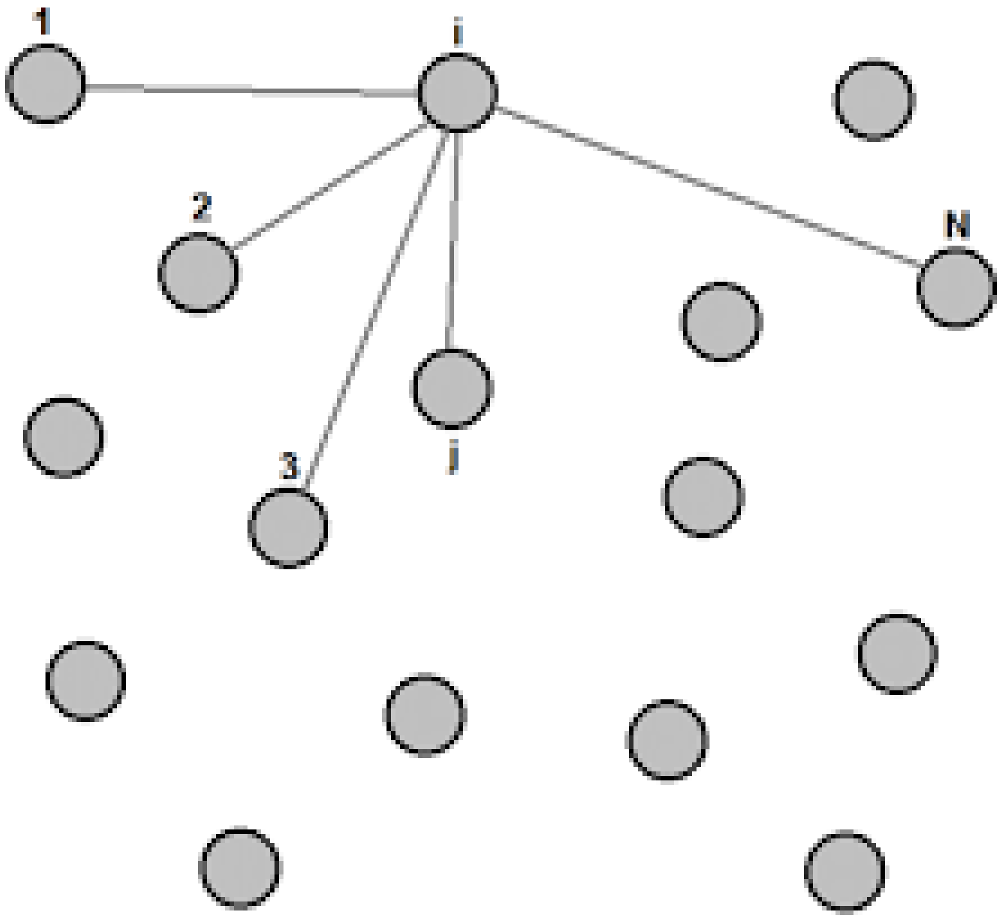
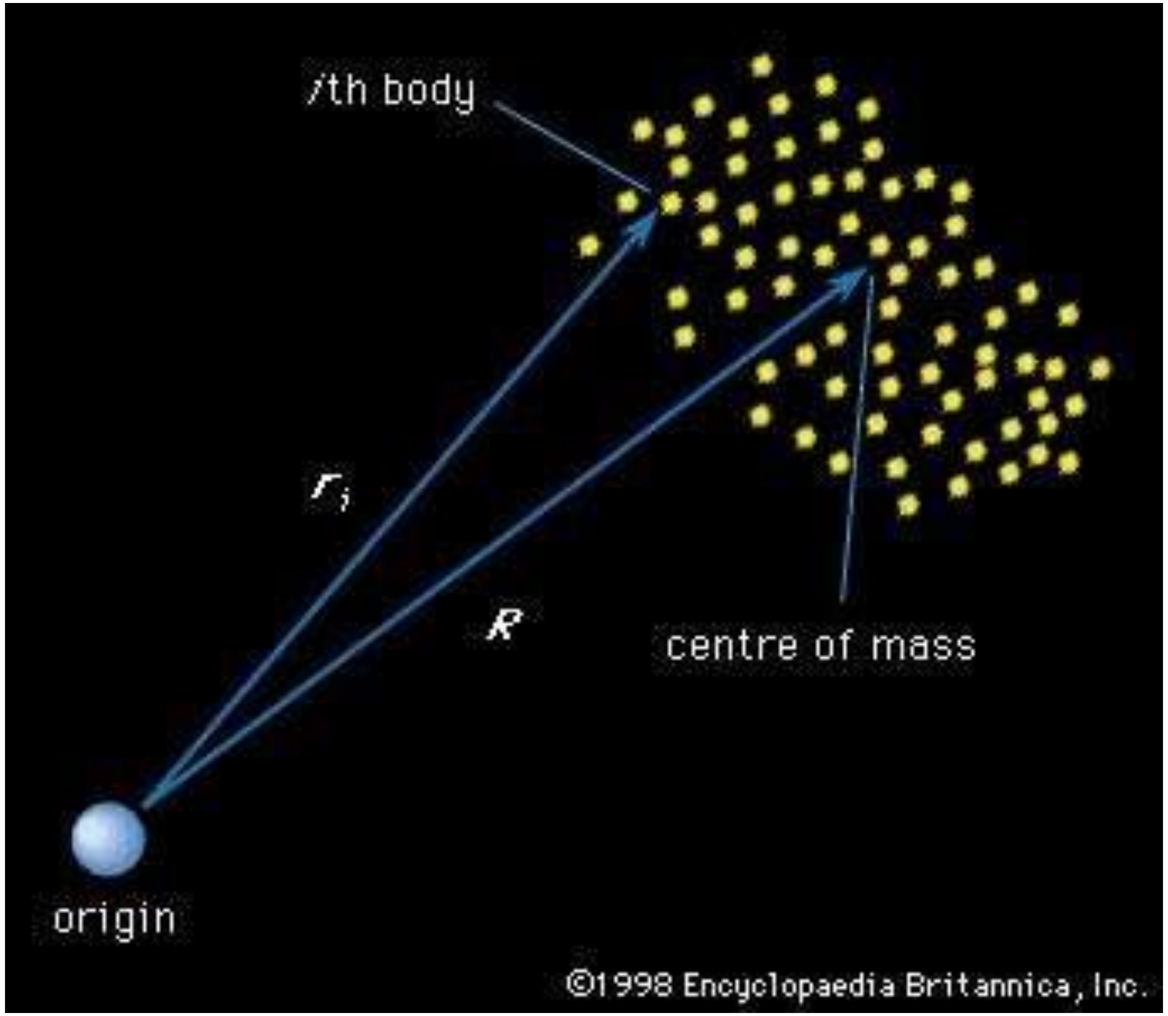
Abstract

- We present an over view of the Hamiltonian of the N-Body problem with some special cases (two- and three-body problems) in view of classical mechanics and General Theory of Relativity. Several applicational models of orbital motions are discussed extensively up to High eccentric problems (Barker's equation, parabolic eccentricity…etc.) by using several mathematical techniques with high accurate computations and perturbation such as: Lagrange's problem, Bessel functions, Gauss method…..etc.
- As an example; we will give an overview of the satellite the position and velocity components of the Jordan first CubeSat: **JY-SAT** which was launched by SpaceX Falcon 9 in December 3, 2018.

N-Body Problem



N-Body Problem with weak gravitational interactions without Dark Matter (galactic dynamics)



- Solving N-Body Problem is an important to understand the motions of the solar system (Sun, Moon, planets), and visible stars, as well as understanding the dynamics of globular cluster star systems

Weak Gravitational Interactions

- The Lagrangain for the N-Body system for the weak gravitational interactions can be written as: (Newtonian Dynamics Limit)

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N} m_i \parallel \dot{q}_i \parallel^2 - \sum_{1\le i<j\le N} \frac{G m_i m_j}{\parallel q_i - q_j \parallel}$$

- Where is q_i is the generalized coordinates,
  $\dot{q}_i$ is related to generalized momenta, and $\mathcal{L}$ is the Lagrangian of the N-Body system.

- Using the principle of least action, the above equation can be written in terms of Euler-Lagrange equations: 3N nonlinear differential equations as:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i}$$

General Relativity orbits (strong gravitational interactions)

The two-body problem in general relativity is the determination of the motion and gravitational field of two bodies as described by the field equations of general relativity EFEs.

- bending of light by gravity,
- the motion of a planet orbiting its sun,
- he motion of binary stars around each other,
- and estimate their gradual loss of energy through gravitational radiation,
- that tidal forces.

General three-Body Problem (Newton to Poincare´)

- In the three-body problem, three bodies move in space under their mutual gravitational interactions as described by Newton’s theory of gravity. Solutions of this problem require that future and past motions of the bodies be uniquely determined based solely on their present positions and velocities. In general, the motions of the bodies take place in three dimensions (3D), and there are no restrictions on their masses nor on the initial conditions. Thus, we refer to this as the general three-body problem.

The three-body problem

- The Lagrangian of the system in Cartesian coordinates:

$$L = \frac{m_1(\dot{\vec{r}}_1)^2}{2} + \frac{m_2(\dot{\vec{r}}_2)^2}{2} + \frac{m_3(\dot{\vec{r}}_3)^2}{2}$$
$$-V\left(\sqrt{(\vec{r}_1 - \vec{r}_2)^2}\right) - V\left(\sqrt{(\vec{r}_1 - \vec{r}_3)^2}\right) - V\left(\sqrt{(\vec{r}_2 - \vec{r}_3)^2}\right)$$

- This problem has 9 independent coordinates entangled by the 3 potential functions

- This Lagrangian cannot be re-gauged to a one-particle Lagrangian

- No general explicit solution is known

Sun-Earth-Moon System

$$\mathbf{F}_{\mathrm{S}} = -G \cdot m_{\mathrm{S}} \cdot \left(\frac{m_{\mathrm{E}}}{|\mathbf{r}_{\mathrm{S}} - \mathbf{r}_{\mathrm{E}}|^3} \cdot (\mathbf{r}_{\mathrm{S}} - \mathbf{r}_{\mathrm{E}}) + \frac{m_{\mathrm{M}}}{|\mathbf{r}_{S} - \mathbf{r}_{M}|^3} \cdot (\mathbf{r}_{\mathrm{S}} - \mathbf{r}_{\mathrm{M}}) \right)$$

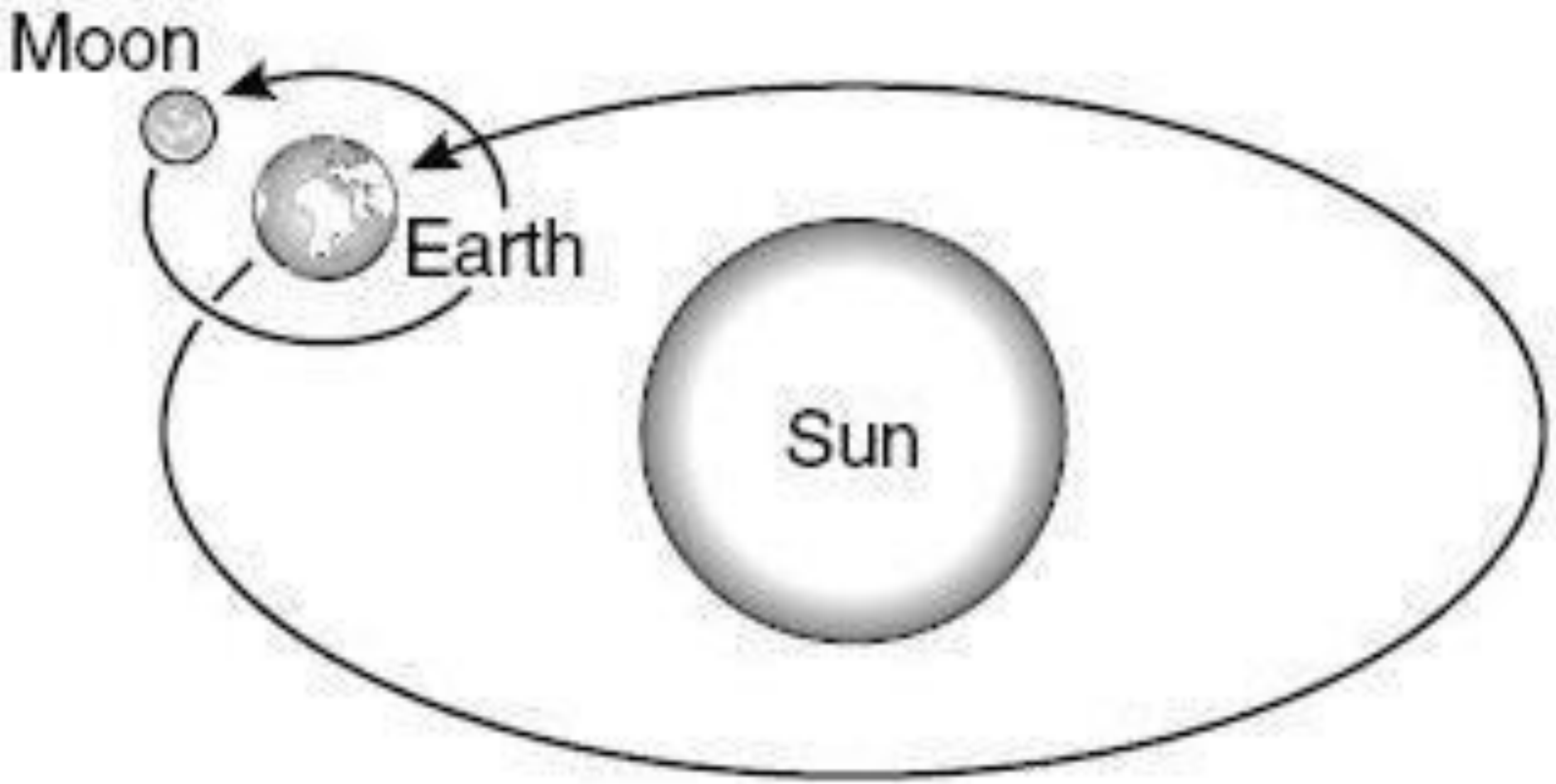
$$\mathbf{F}_{\mathrm{E}} = -G \cdot m_{\mathrm{E}} \cdot \left(\frac{m_{\mathrm{S}}}{|\mathbf{r}_{\mathrm{E}} - \mathbf{r}_{\mathrm{S}}|^3} \cdot (\mathbf{r}_{\mathrm{E}} - \mathbf{r}_{\mathrm{S}}) + \frac{m_{\mathrm{M}}}{|\mathbf{r}_{\mathrm{E}} - \mathbf{r}_{\mathrm{M}}|^3} \cdot (\mathbf{r}_{\mathrm{E}} - \mathbf{r}_{\mathrm{M}}) \right)$$

$$\mathbf{F}_{\mathrm{M}} = -G \cdot m_{\mathrm{M}} \cdot \left(\frac{m_{\mathrm{S}}}{|\mathbf{r}_{\mathrm{M}} - \mathbf{r}_{\mathrm{S}}|^3} \cdot (\mathbf{r}_{\mathrm{M}} - \mathbf{r}_{\mathrm{S}}) + \frac{m_{E}}{|\mathbf{r}_{\mathrm{M}} - \mathbf{r}_{\mathrm{E}}|^3} \cdot (\mathbf{r}_{\mathrm{M}} - \mathbf{r}_{\mathrm{E}}) \right)$$

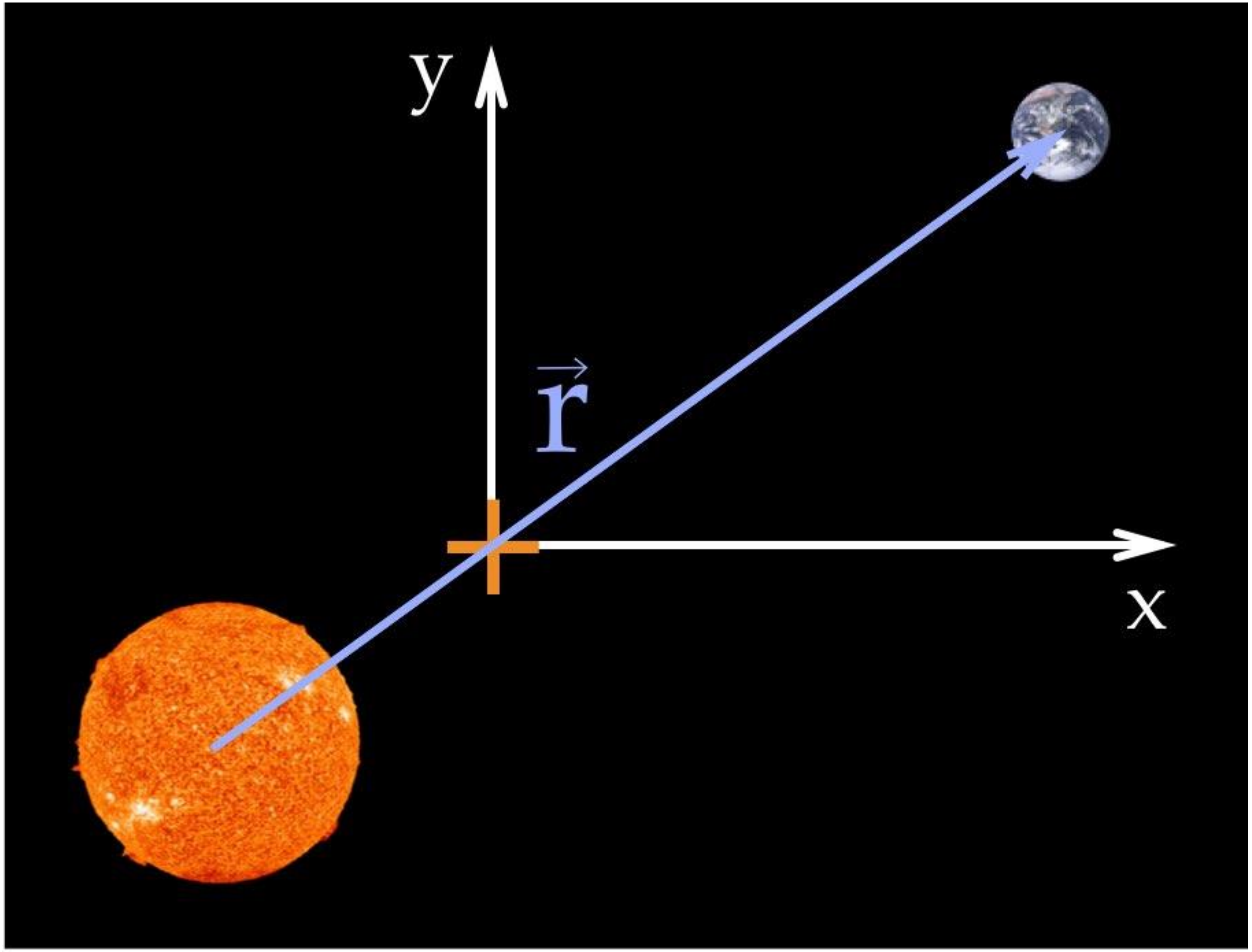
$$\ddot{\mathbf{r}}_1 = -Gm_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - Gm_3 \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3},$$

$$\ddot{\mathbf{r}}_2 = -Gm_3 \frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - Gm_1 \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3},$$

$$\ddot{\mathbf{r}}_3 = -Gm_1 \frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - Gm_2 \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}.$$


Moon
Earth
Sun

Two Body Problem



- The two body problem involves a pair of particles with masses m1 and m2 described by a Lagrangian of the form

$$L = \frac{1}{2} m_1 \dot{\vec{r}}_1^2 + \frac{1}{2} m_2 \dot{\vec{r}}_2^2 - V(|\vec{r}_1 - \vec{r}_2|).$$

- Relative motion (CM)

$$L = \frac{1}{2} m \dot{\vec{r}}^2 - V(|\vec{r}|),$$

$$L(r, \dot{r}, \dot{\phi}) = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\phi}^2) - V(r).$$

$$\phi - \phi_0 = \int_{r_0}^{r} \frac{L}{x^2 \sqrt{2m(E - V(x)) - \frac{L^2}{x^2}}} dx.$$

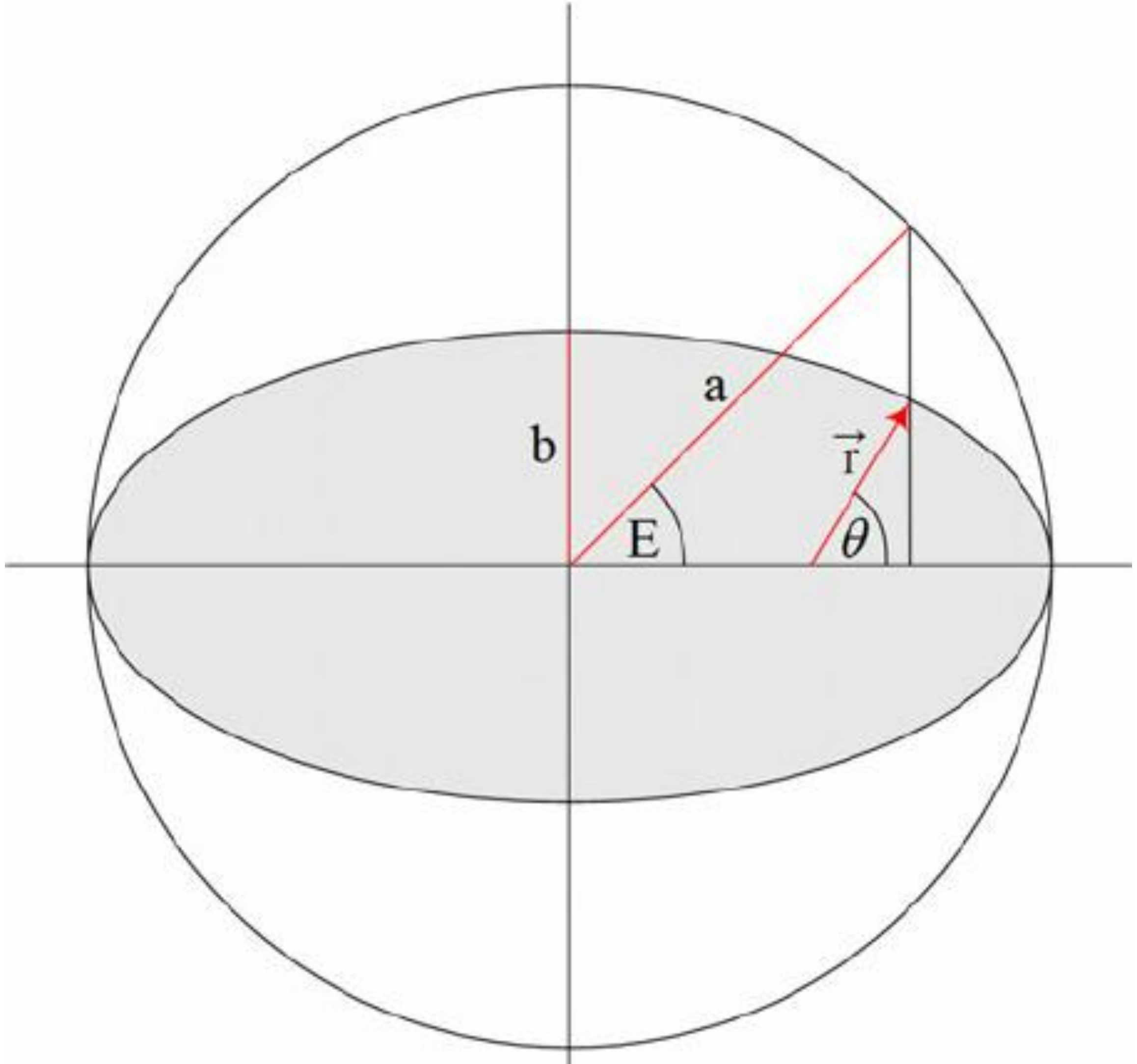
$$t - t_0 = \pm \int_{r_0}^{r} \frac{1}{\sqrt{\frac{2}{m}(E - V(x)) - \frac{L^2}{m^2 x^2}}} dx.$$

Two Body Problem
Kepler's Equation

One of the problems in the celestial mechanics is the solution of Kepler's equation (KE), the literature of solving this equation is an extensive, and widely studied over three centuries, from Newtron's days up to now (Colwell 1993).

Geometric Representation

- Where
- E is the Eccentric anomaly
- θ is the True anomaly
- b is the seminar axes
- a is the semimajor axes
- r is the radial position



- The universal form of Kepler's equation (UKE) can be written as:

$$\sqrt{\mu}t = \frac{\vec{R}_0 \bullet \vec{V}_0}{\sqrt{\mu}} x^2 C(\alpha x^2) + (1 - \left|\vec{R}_0\right|) x^3 S(\alpha x^2) + \left|\vec{R}_0\right| x$$

- where *μ* is the gravitational constant, *R_0 and V_o are the position and velocity at t=0, α=1/a (semimajor axis), e eccentricity, and C(x) and S(x) are defined as*

$$C(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{n-1}}{(2n)!} = \begin{cases} \frac{1-\cos\sqrt{x}}{x} & x < 0 \\ \\ \frac{\cosh\sqrt{-x}-1}{-x} & x > 0 \end{cases}$$

$$S(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{n-1}}{(2n+1)!} = \begin{cases} \frac{\sqrt{x}-\sin\sqrt{x}}{(\sqrt{x})^3} & x > 0 \\ \\ \frac{\sinh\sqrt{-x}-\sqrt{-x}}{(\sqrt{-x})^3} & x < 0 \end{cases}$$

$$x = \frac{E}{\alpha^{\frac{1}{2}}} = \sqrt{a}E \ \ and \ M = \sqrt{\frac{\mu}{a^3}}t$$

$$M = E - e\sin E$$

Kepler's Equation

$$M = E - e \sin E$$

$$r = a(1 - e \cos E)$$

$$\tan(\frac{v}{2}) = \sqrt{\frac{1+e}{1-e}} \tan(\frac{E}{2})$$

- Where M is the mean anomaly (related to time parameter). e is the eccentricity, and E is the eccentric anomaly, in general M and e are known and E is to be calculated.
- M=(t-T)(2Pi/P) (T the time of perihelion passage)
- Radius vector r
- True anomaly v

Bessel's function Solution of KE

$$E = M + \sum_{k=1}^{\infty} \frac{2}{k} J_k(ke) \sin(kM)$$

- Where J_k is the Bessel function of the first kind

An Iterative Mothed

Make a first approximation, E_0=M, and substitute into rhs of Kepler's equation to get a better approximation. Keep doing this to any order.
For the first six order, we get:

$$M + 2e\sin(M) + e^2\sin(2M) + 2e^3\{-\frac{\sin(M)}{8} + \frac{3}{8}\sin(3M)\} + 2e^4\{-\frac{1}{6}\sin(2M) + \frac{1}{3}\sin(4M)\}$$
$$+2e^5\{\frac{\sin(M)}{192} - \frac{27}{128}\sin(3M)\} + \frac{125}{384}\sin(5M)\} + 2e^6\{\frac{1}{48}\sin(2M) - \frac{4}{15}\sin(4M) + \frac{27}{80}\sin(6M)\}$$

- Which is good approximation for small e

Newton's Iteration Function

$$g(E) = E - \frac{F(E)}{F'(E)}$$

$$F(E) = E - M - e\sin(E)$$

$$E_{n+1} = E_n - \frac{F(E_n)}{F'(E_n)} = E_n - \frac{E_n - e\sin E_n - M}{1 - e\cos E_n}.$$

Example: A satellite moves on an elliptic orbit with ellipticity $e = 0.00001$. The mean anomaly is $30°$, Find an approximate value of the eccentric anomaly. In this case $M = \pi/6 = .5235988$. We set

$E_1 = M$. Using the iteration we get

$$\begin{aligned} E_2 &= M + e \sin E_1 = .5235988 + 0.00001 \sin(.5235988) = .5236038 \\ E_3 &= M + e \sin E_2 = .5235988 + 0.00001 \sin(.5236038) = .5236038. \end{aligned}$$

We see that after only two iterations, we get $E_3 = E_2$ and the approximate value of the eccentric anomaly is $.5236038$ or $30.00028°$. The method converges not always that fast, depending on the values of e and M. For larger values of e, we can also use Newton's method. For this we rewrite the equation as

Laguerre's Iterative Function

- The laguerre's iterative function (Conway 1986, Danby 1992) can be written as:

$$L(E) = \frac{N(E - M - e\sin(E))}{(1 - e\cos(E)) \pm \sqrt{|(N-1)^2(1 - e\cos(E))^2 - N(N-1)(e\sin(E))(E - M - e\sin(E)|}} \quad (12)$$

- Where

$$f(E) = E - M - e\sin(E)$$

$$f'(E) = 1 - e\cos(E)$$

$$f''(E) = e\sin(E)$$

Laguerre's Iterative Function

- For a simple iteration of E0=M= 0.09424777960769381 it will be converge to 0.8298940924910203 after 97 iterations.
- And for E0=M= 0.09424777960769 , it will be converge 0.8298940924910086 after 97 iterations
- Here we use Laguerre’s method iteration up to 16th decimal places. For E0=M= 0.09424777960769381, it is shown that the convergence in 4 iterations.
- For E0=M=0.09424777960769, it is shown that the convergence in 4 iterations.
- Reference

- **A. Sakaji, Electronic Journal of Theoretical Physics, 1, 15-29, 2004**

No	E
1	2.7312188798849880
2	−742.4948117613755100
3	659.1835488857170084
4	−3723.4179211332550526
5	−1660.4734076425097380
6	−205.7127960209346314
7	−10.8553113652700991
8	−0.3788018096213978
9	1.0374990945997747
10	0.8657475853242108
11	0.8312590206737048
12	0.8298961829698286
13	0.8298940924959349
14	0.8298940924910203
15	0.8298940924910203

No	E
1	12.7312188798850832
2	−742.4948117606629964
3	659.1835505445485100
4	−3723.4431303868830369
5	−1643.1019303801188896
6	−819.7523311038079860
7	−405.0480343702568532
8	−199.8687017980007438
9	117.7712090225128621
10	3.4854679090025988
11	1.5627982792317355
12	1.0874252820882762
13	0.8816949332987970
14	0.8326736588297812
15	0.8299027416878744
16	0.8298940925751370
17	0.8298940924910087
18	0.8298940924910087

- we see that **Laguerre's method** is repaid convergent, it takes 4 iterations, but Newton's method takes 15 to 18 iterations.

No	E
1	1.19014859638984185
2	0.83774121975450705
3	0.82989415677133444
4	0.82989409249102031
5	0.82989409249102031

No	E
1	1.1901485963898382
2	0.8377412197544960
3	0.8298941567713228
4	0.8298940924910087
5	0.8298940924910087

The extreme example e=0.999999999 and M=0.00000001, by Laguerre's method we have

No	E
1	1.089327272448603612
2	0.431829495877395164
3	0.160692406060242765
4	0.059094666058560924
5	0.021721546671943329
6	0.008174602667263537
7	0.004174906422037452
8	0.003914412222418555
9	0.003914357769065121
10	0.003914357768951919
11	0.003914357768951919

- And for e=0.9 amd M=1 radian, Charles and Tatum (1998) showed the subsequent iterations and convergence to 1.862086686874532 after **20** iterations. Let us try this again by Laguerre's method (3 iterations).

No	E
1	1.8461295788421068
2	1.8620863712062943
3	1.8620866868745323

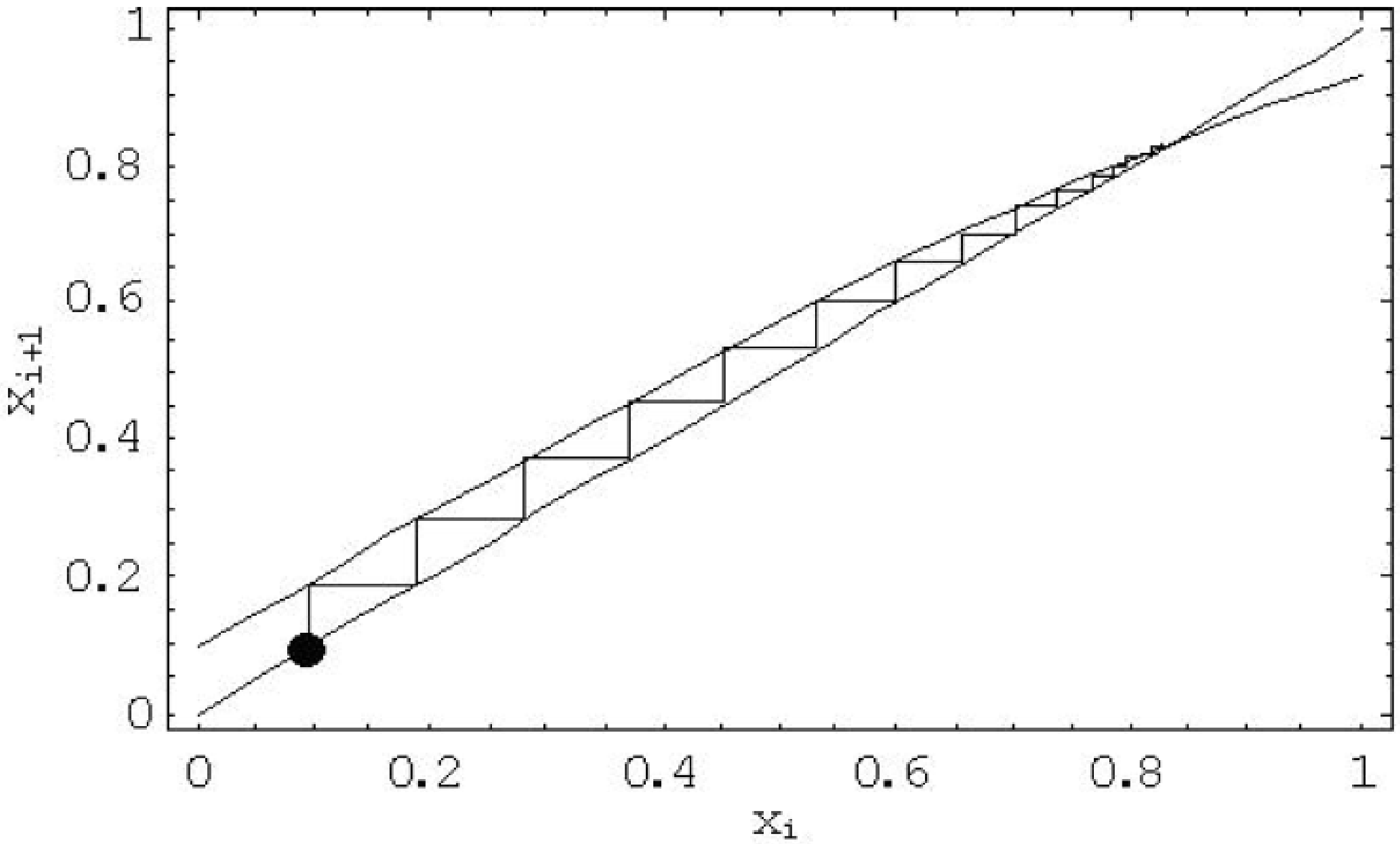


Fig. 1 Cobweb plot for E_o=M= 0.0942477796076938l and e=0.997

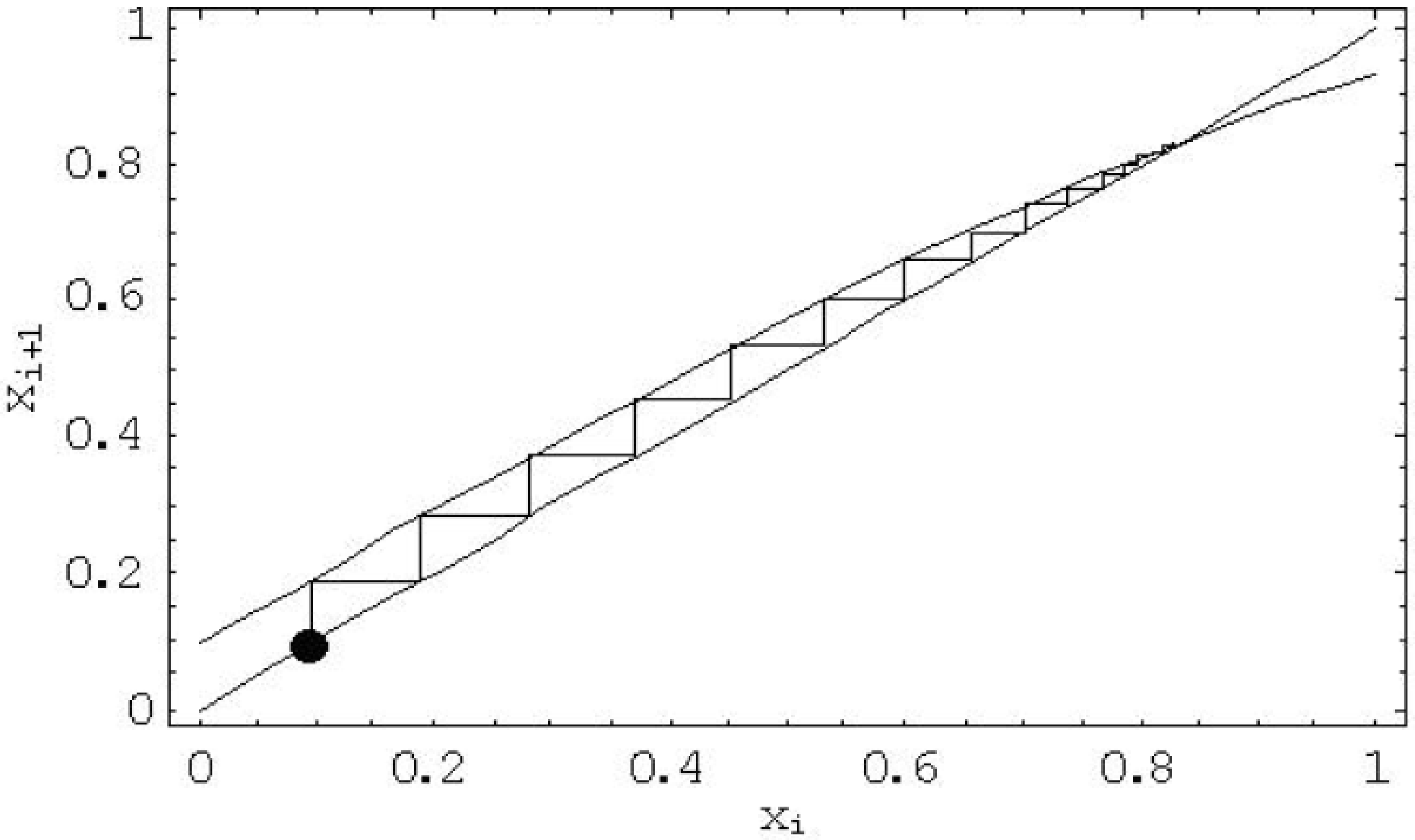


Fig. 2 Cobweb plot for E_o=M= 0.09424777960769 and e=0.997

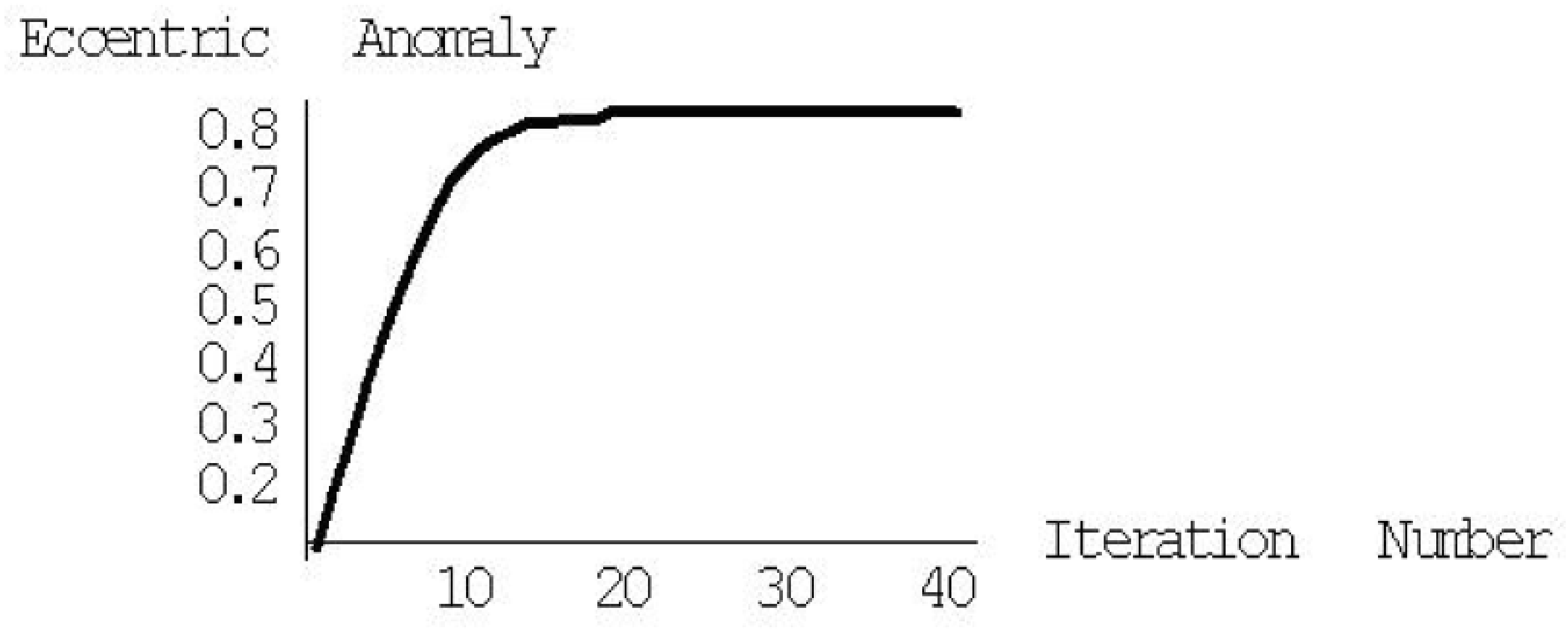


Fig. 3 E versus Iteration number ,for E_o=M= 0.0942477796076938 and e=0.997

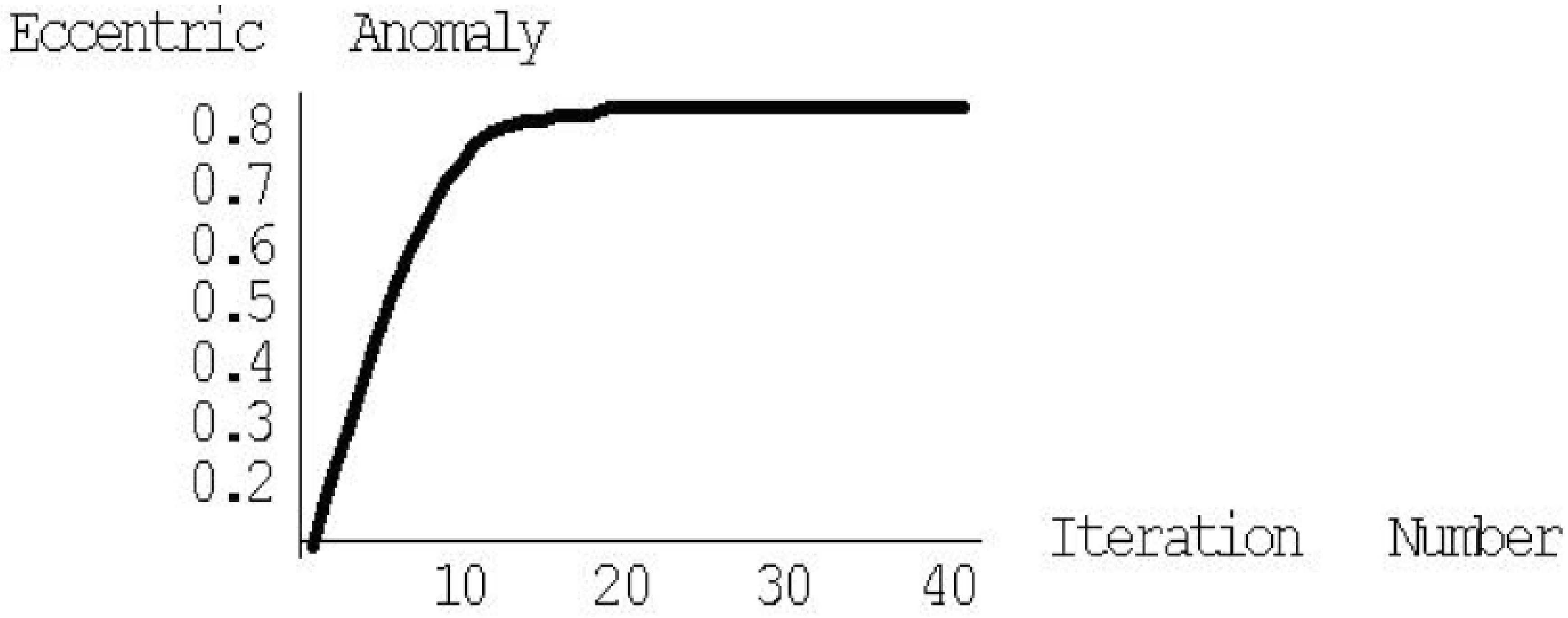


Fig. 4 E versus Iteration number , for E_o=M= 0.09424777960769 and e=0.997

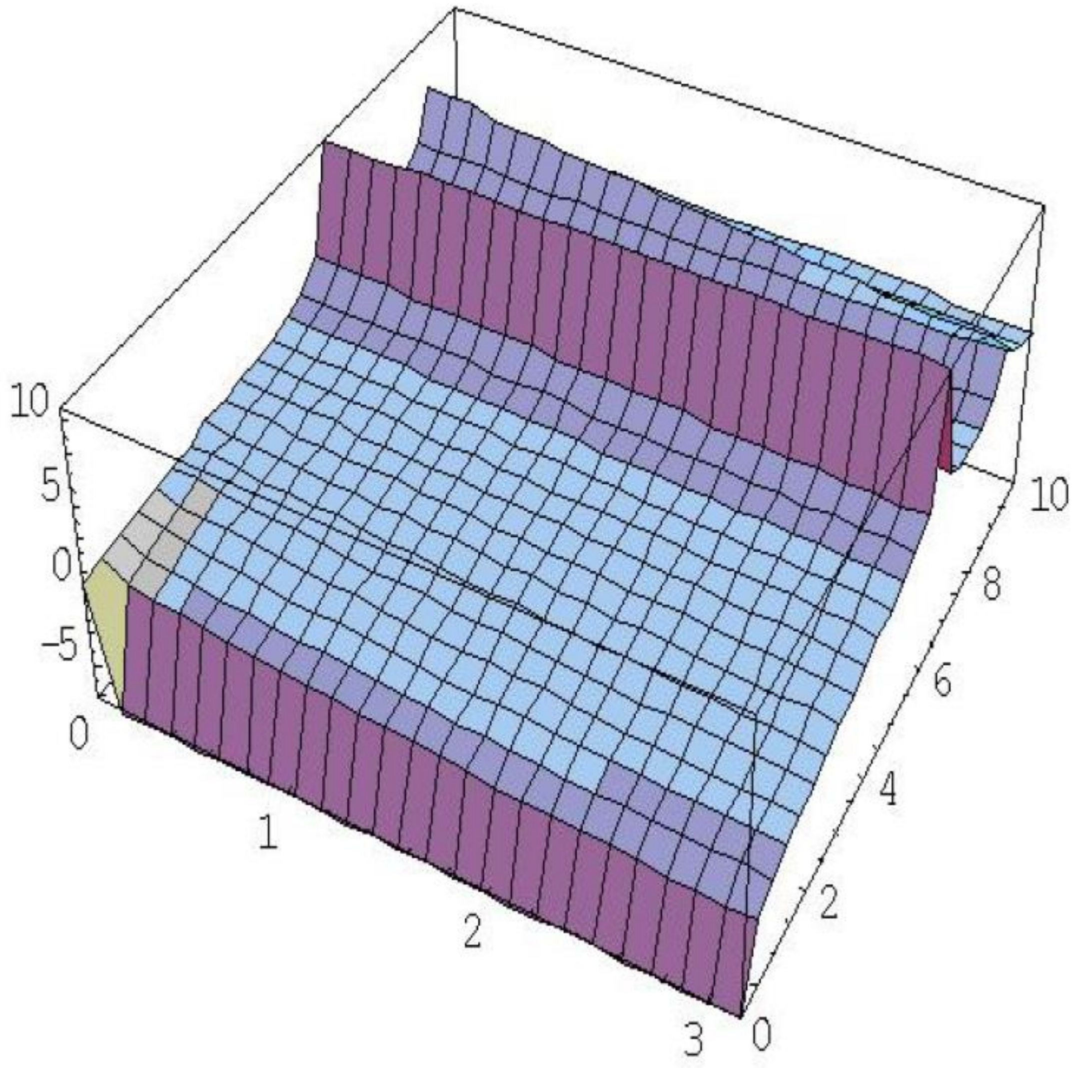


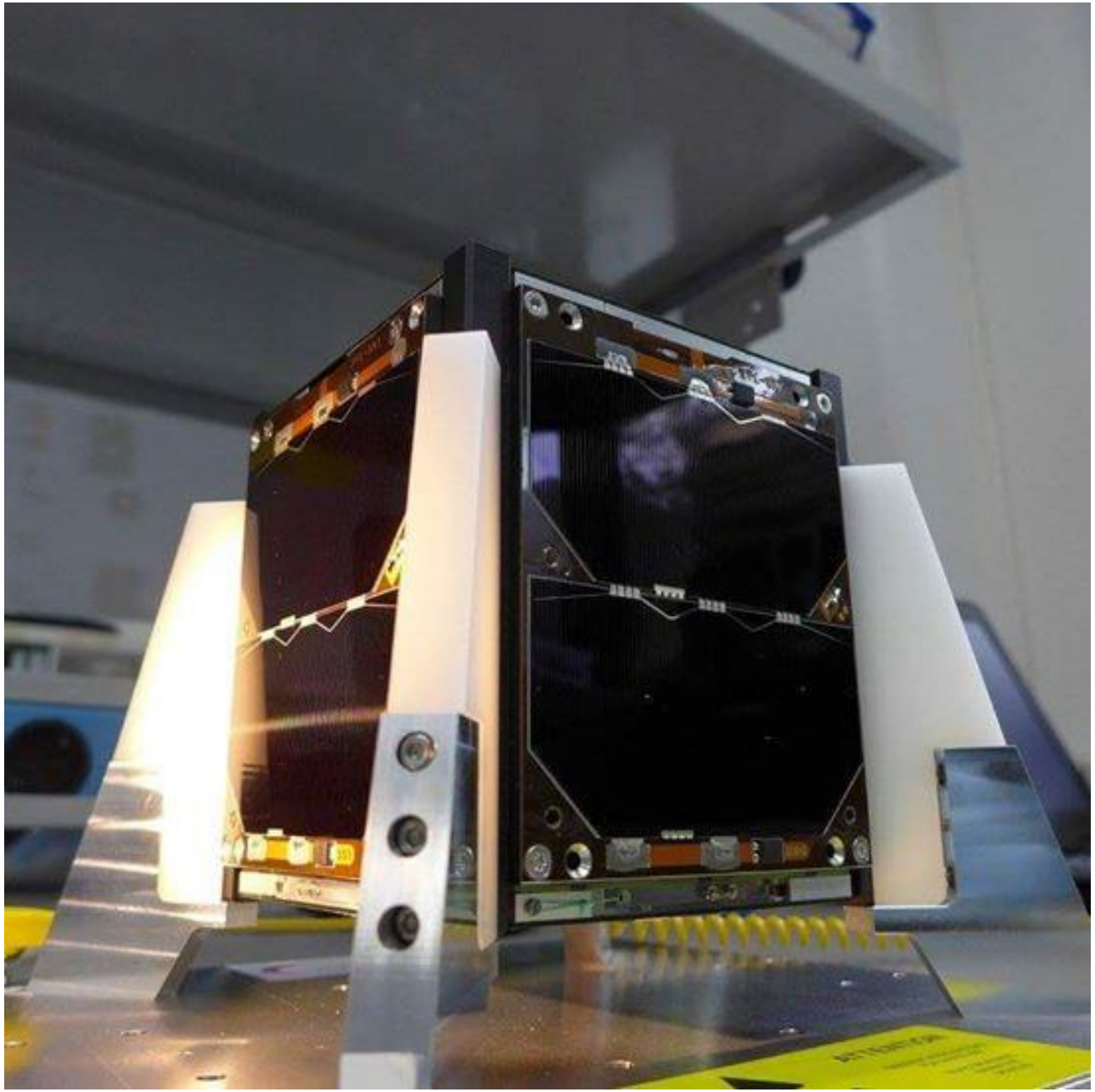
Fig. 7 Laguerre's iteration function for for e=0.997 M=(0-3) and E=(0-10)

0	0	0	0
0.1	0.6308435275631535	0.8316604237910568	0.8535302901646385
0.2	0.9112350046181908	1.066997365281563	1.083525742942339
0.3	1.103517720303087	1.234564589808617	1.248376640429232
0.4	1.255685962364815	1.370135782880695	1.382163246392088
0.5	1.384412720202163	1.486483282761429	1.497192750166678
0.6	1.497589413390409	1.589820848846042	1.599488450417671
0.7	1.599626008167092	1.683697135888974	1.692503553021586
0.8	1.693259659658157	1.770353009526324	1.778424700470779
0.9	1.780317547935176	1.851305464552152	1.858735309303670
1.	1.862086686874532	1.927635550695835	1.934494276402446
1.1	1.939511930562878	2.000144791246475	2.006487781487473
1.2	2.013310231004202	2.069446577778808	2.075318169739842
1.3	2.084040546877669	2.136022700810168	2.141459023579514
1.4	2.152148684681210	2.200259957380998	2.205290877585482
1.5	2.217997202598577	2.262474771378084	2.267125268622691
1.6	2.281886016436808	2.322930287520673	2.327221452229856
1.7	2.344067001628836	2.381848567596671	2.385798340984930
1.8	2.404754595260939	2.439419500821278	2.443043234001429
1.9	2.464133660601510	2.495807451134292	2.499118334370725
2.	2.522365434000245	2.551156310065828	2.554165706830331
2.1	2.579592101007881	2.605593403861859	2.608311113770348
2.2	2.635940375246179	2.659232563079221	2.661667030766210
2.3	2.691524340708805	2.712176570873782	2.714335053093388
2.4	2.746447743038013	2.764519144640187	2.766407844576619
2.5	2.800805864303132	2.816346563655770	2.817970739089150
2.6	2.854687080566958	2.867739026237677	2.869103076528127
2.7	2.908174176831871	2.918771799416825	2.919879335035356
2.8	2.961345476456292	2.969516209545751	2.970370106957870
2.9	3.014275829638193	3.020040511809478	3.020642955813272
3.	3.067037496630689	3.070410669117502	3.070763184187473
3.1	3.119700955021393	3.120691065529710	3.120794537274652

Fig. 14 This table shows the solutions of Kepler's equation for some extreme values (e=0.9, 0.99, 0.9999) respectively.

JY1SAT

JORDAN'S FIRST SATELLITE



- Jordan’s Crown Prince Al Hussein bin Abdullah II announced on Monday, December 3, the launch of the first Jordanian satellite into space, as part of the Masar initiative by the Crown Prince Foundation.
- “The first Jordanian cube satellite, JY1, was launched from California into space today. A great achievement and a major first step in satellite engineering for the youth of Jordan.”

- JY1 was built by several Jordanian students, academics and consultants, who explored satellite engineering and related knowledge, and developed their skills in aviation and space science through an intensive program over a period of 10 weeks, under the supervision of NASA experts.
- The satellite aims to carry out research and academic tasks, as well as take and publish promotional pictures of Jordan’s historical and folkloric sites.
- The nano cube satellite will fly over Jordan and take SSTV images and various data of the Kingdom, and the information it releases will be available to everyone via two platforms: a mobile application for amateur use, and a more advanced desktop software.

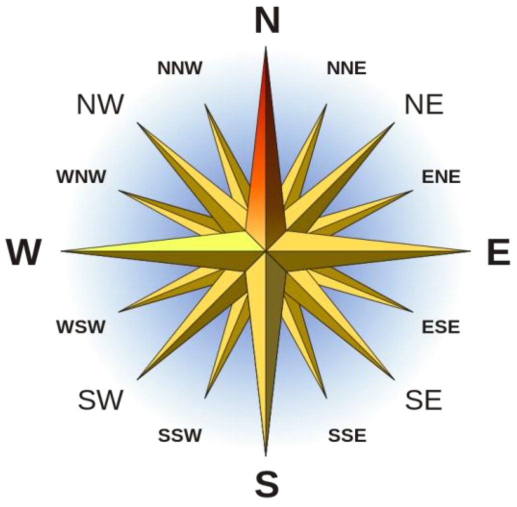
Two-line element set of Jordan cube satellite (JY1SAT)

A two-line element set (TLE) is a data format encoding a list of orbital elements of an Earth-orbiting object for a given point in time, the epoch of JY1SAT, Using Kepler's equation with some corrections (perturbations models), the state (position and velocity) at any point in the past or future can be estimated to some accuracy

- JY1-SAT has been given the NORAD ID 43803 and it has the following TLEs (Source of the Keplerian elements: AFSPC) :
- **1. 43803U 18099AX 19035.19214779 .00000222 00000-0 25484-4 0 9996**
- **2. 43803 97.7578 108.7107 0016371 60.9685 299.3174 14.95165049 9322**
- NORAD ID: 43803
- Int'l Code: 2018-099AX
- Perigee: 579.3 km
- Apogee: 598.5 km
- Inclination: 97.8 °
- Period: 96.3 minutes
- Semi major axis: 6959 km
- Launch date: December 3, 2018

JY1SAT: Azimuth and Elevation angle (visible above Amman: Good to Excellent)

Start		Max Altitude			End	
Date, Local time	AZ	Local time	Az	El	Local time	Az
29-Mar 22:10	SSE 155	22:17	ENE 73	50	22:23	N 356
30-Mar 22:16	SSE 159	22:22	E 81	59	22:28	N 354
31-Mar 22:21	SSE162	22:27	E 78	70	22:34	N 352
1–Apr 22:27	S 165	22:33	ENE 63	83	22:39	N 350



All Passes above Amman

Start		Max altitude			End	
Date, Local time	Az	Local time	Az	El	Local time	Az
23-Mar 20:39	SE 133°	**20:44**	**ENE 72°**	**20°**	20:49	N 9°
23-Mar 22:14	SSW 196°	**22:19**	**W 266°**	**24°**	22:25	NNW 331°
24-Mar 09:59	N 15°	**10:05**	**ESE 105°**	**68°**	10:11	S 187°
24-Mar 20:44	SE 137°	**20:49**	**ENE 74°**	**23°**	20:55	N 7°
24-Mar 22:19	SSW 198°	**22:25**	**W 264°**	**21°**	22:30	NW 328°
25-Mar 10:05	N 13°	**10:11**	**E 79°**	**80°**	10:16	S 190°
25-Mar 20:49	SE 141°	**20:55**	**ENE 69°**	**27°**	21:00	N 5°
25-Mar 22:25	SSW 202°	**22:30**	**W 268°**	**18°**	22:35	NW 326°
26-Mar 10:10	N 11°	**10:16**	**SW 229°**	**83°**	10:22	S 194°
26-Mar 20:55	SE 144°	**21:00**	**ENE 72°**	**31°**	21:06	N 2°
26-Mar 22:30	SSW 206°	**22:36**	**W 265°**	**15°**	22:41	NW 323°
27-Mar 10:16	N 9°	**10:22**	**W 282°**	**73°**	10:27	SSW 198°
27-Mar 21:00	SE 148°	**21:06**	**E 76°**	**36°**	21:11	N 0°
27-Mar 22:36	SSW 210°	**22:41**	**W 269°**	**13°**	22:46	NW 321°
28-Mar 10:21	N 7°	**10:27**	**WNW 292°**	**61°**	10:33	SSW 201°
28-Mar 21:05	SSE 151°	**21:11**	**ENE 68°**	**42°**	21:17	N 358°
28-Mar 22:42	SW 216°	**22:47**	**W 267°**	**11°**	22:51	NW 318°

29-Mar 10:26	N 5°	**10:33**	**W 278°**	**51°**	10:38	SSW 205°
29-Mar 21:11	SSE 155°	**21:17**	**ENE 73°**	**50°**	21:22	N 356°
30-Mar 08:58	NE 41°	**09:02**	**E 91°**	**10°**	09:07	SE 140°
30-Mar 10:32	N 3°	**10:38**	**W 285°**	**43°**	10:44	SSW 208°
30-Mar 21:16	SSE 158°	**21:22**	**E 81°**	**59°**	21:28	N 354°
31-Mar 09:03	NE 39°	**09:08**	**E 94°**	**12°**	09:13	SE 147°

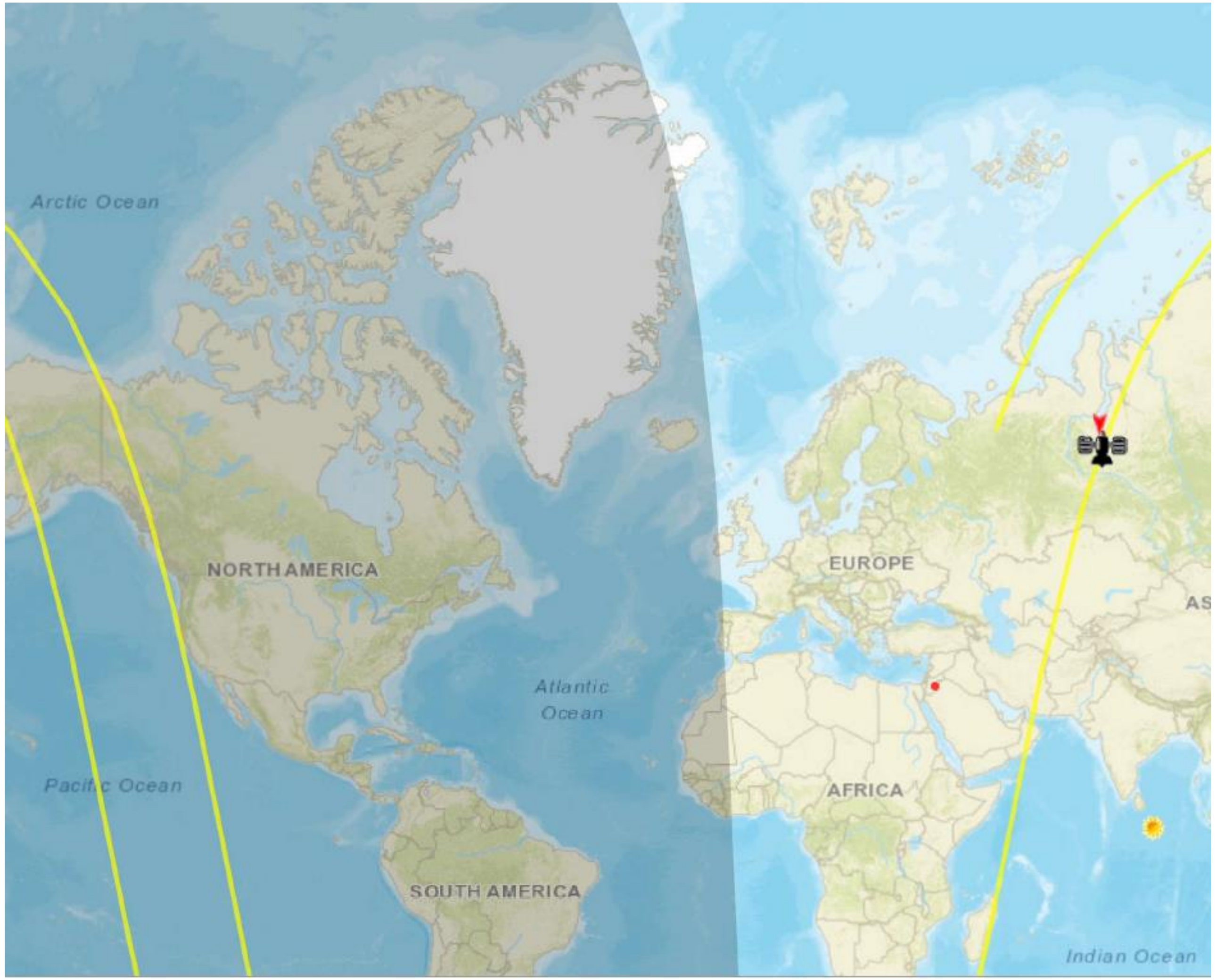
31-Mar 21:21	SSE 162°	**21:28**	**NE 58°**	**69°**	21:33	N 352°
1-Apr 09:08	NE 36°	**09:13**	**E 92°**	**15°**	09:18	SSE 151°
1-Apr 10:43	N 359°	**10:49**	**WNW 292°**	**31°**	10:54	SW 216°
1-Apr 21:27	S 165°	**21:33**	**ENE 63°**	**83°**	21:39	N 350°
2-Apr 09:14	NE 34°	**09:19**	**E 96°**	**17°**	09:24	SSE 155°

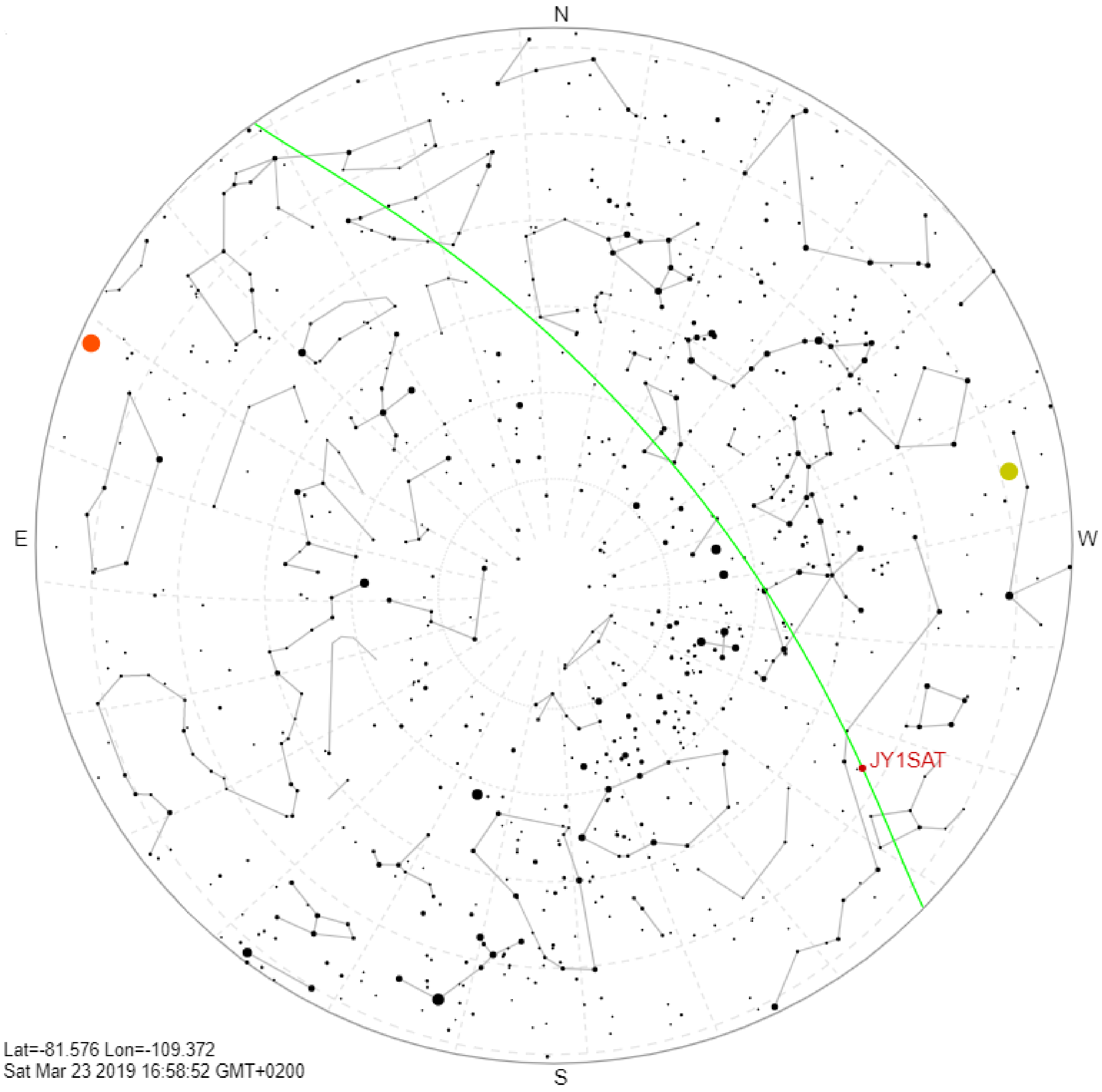
March 23, 16:20:49

- NORAD ID:43803
- LOCAL TIME:16:20:49
- LATITUDE:49.09
- LONGITUDE:-51.41
- ALTITUDE [km]:595.45
- SPEED [km/s]:7.56=27,216km/h
- AZIMUTH:313.7NW
- ELEVATION:-28.4
- RIGHT ASCENSION:19h 31m 51s
- DECLINATION:15° 16' 30''
- Local Sidereal Time:04h 47m 45s
- The satellite is in day light
- SATELLITE PERIOD:97 min

March 26, 08:30:53

- NORAD ID:43803
- LOCAL TIME:08:30:53
- LATITUDE:65.85
- LONGITUDE:-74.66
- ALTITUDE [km]:600.05
- SPEED [km/s]:7.56=27,216km/h
- AZIMUTH:23 NNE
- ELEVATION:-13.7
- RIGHT ASCENSION:07h 09m 56s
- DECLINATION:39° 18’ 57''
- Local Sidereal Time:21h 08m 21s
- The satellite is in day light
- SATELLITE PERIOD:97 min





Lat=-81.576 Lon=-109.372
Sat Mar 23 2019 16:58:52 GMT+0200

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thank you
danke
謝謝
ngiyabonga
شكراً جزيلاً
рахмат
teşekkür ederim
Баярлалаа
спасибо
mersi
kia ora
barka
welalin
tack
spas
faafetai lava
vinaka
спасибі
blagodaram
dank je
misaotra
matondo
paldies
grazzi
mahalo
gracias
tapadh leat
хвала
asante
manana
obrigada
tenki
murakoze
kiitos
dankie
nanni
nandri
dhanyavad
bedankt
hvala
mauruuru
köszönöm
bayarlalaa
gràcie
enkosi
dziękuję
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djiere dieuf
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mochchakkeram
дякую
mamnun
chokrane
chnorakaloutioun
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sulpáy
go raibh maith agat
sobodi
děkuji
obrigado
sagolun
sukriya
kop khun krap
taiku
arigatô
takk
dakujem
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mèsi
didi madloba
najis tuke
ありがとう
tanemirt
grazie
rahmet
ευχαριστώ
diolch
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kam sah hamnida
rahmat
terima kasih
xièxie
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